

# **Test Bank for Calculus Early Transcendentals 12th Anton**

1. Find the average rate of change of  $y$  with respect to  $x$  for  $y = f(x) = \frac{4}{x^2}$  over the interval  $[1, 8]$ .

A) 0.438    B) -0.563    C) -0.580    D) -27.562    E) 3.938

Ans: B

Difficulty: Easy

Section: 2.1

2. Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[2, 7]$ .  $y = f(x) = 2x^3$

A) 74    B) 140    C) 134    D) 96    E) 335

Ans: C

Difficulty: Easy

Section: 2.1

3. Find the instantaneous rate of change of  $y = 3x^2$  with respect to  $x$  at  $x_0 = 5$ .

A) 6    B) 30    C) 10    D) 15    E) 16

Ans: B

Difficulty: Easy

Section: 2.1

4. Find the instantaneous rate of change of  $y = \frac{7}{x}$  with respect to  $x$  at  $x_0 = 5$ .

A) -175    B) -6.9600    C) 0.2800    D) -0.2800    E) -0.1020

Ans: D

Difficulty: Medium

Section: 2.1

5. Find the instantaneous rate of change of  $y = 11x^5$  with respect to  $x$  at a general point  $x_0$ .

A)  $55x_0^5$     B)  $11x_0$     C)  $11x_0^5$     D)  $11x_0^4$     E)  $55x_0^4$

Ans: E

Difficulty: Easy

Section: 2.1

6. Find the instantaneous rate of change of  $y = \frac{-6}{x^3}$  with respect to  $x$  at a general point  $x_0$ .

A)  $\frac{18}{x_0^3}$     B)  $\frac{-6}{x_0^4}$     C)  $\frac{18}{x_0^4}$     D)  $\frac{-18}{x_0^4}$     E)  $\frac{-18}{x_0^3}$

Ans: C

Difficulty: Medium

Section: 2.1

7. Find the slope of the tangent line to the graph of  $f(x) = 5x^5 - 7$  at a general point  $x_0$ .

A)  $25x_0^4 - 7$    B)  $5x_0^4$    C)  $25x_0^4$    D)  $5x_0^4 - 1$    E)  $5x_0^4 - 7$

Ans: C

Difficulty: Easy

Section: 2.1

8. Answer true or false. The slope of the tangent line to the graph of  $f(x) = 2x^2 - 1$  at  $x_0 = 5$  is 19.

Ans: False

Difficulty: Easy

Section: 2.1

9. Answer true or false. Use a graphing utility to graph  $y = 4t^3$  on  $[0, 3]$ . If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is increasing over the graphed domain.

Ans: True

Difficulty: Easy

Section: 2.1

10. Use a graphing utility to graph  $y = t^2 - 5t + 4$  on  $[0, 10]$ . If this graph represents a position versus time curve for a particle, the instantaneous velocity of the particle is zero at what time? Assume time is in seconds.

A) 4.5s   B) 3s   C) 2.5s   D) 1.5s   E) 5s

Ans: C

Difficulty: Medium

Section: 2.1

11. A rock is dropped from a height of 828.1 meters and falls toward earth in a straight line. In  $t$  seconds the rock drops a distance of  $4.9t^2$  meters. What is the instantaneous velocity downward when it hits the ground?

A) 3,360,173.089 meters/s   D) 9.8 meters/s  
B) 127.4 meters/s   E) 26 meters/s  
C) 63.7 meters/s

Ans: B

Difficulty: Easy

Section: 2.1

12. Answer true or false. The magnitude of the instantaneous velocity is always less than the magnitude of the average velocity.

Ans: False

Difficulty: Easy

Section: 2.1

13. Answer true or false. If a rock is thrown straight upward to a height of 25 feet from the ground, when it returns to earth its average velocity will be its initial velocity.

Ans: False

Difficulty: Easy

Section: 2.1

14. Answer true or false. If an object is thrown straight upward with an instantaneous velocity of 20 m/s, its instantaneous velocity at the point where it stops rising is 0.

Ans: True

Difficulty: Easy

Section: 2.1

15. An object moves in a straight line so that after  $t$  s its distance in mm from its original position is given by  $s = 3t^3 + 4t$ . Its instantaneous velocity at  $t = 3$  s is

A) 81 mm    B) 247 mm    C) 733 mm    D) 85 mm    E) 31 mm

Ans: D

Difficulty: Medium

Section: 2.1

16. Find the instantaneous rate of change of  $y$  with respect to  $x$  at  $x_0 = 4$ .  $y = 4x^2 - 8$

A) 32    B) 24    C) 16    D) 40    E) 64

Ans: A

Difficulty: Easy

Section: 2.1

17. Find the instantaneous rate of change of  $y$  with respect to  $x$  at  $x_0 = 49$ .  $y = \sqrt{x} - 6$

A)  $\frac{1}{14}$     B)  $\frac{1}{7}$     C)  $\frac{11}{7}$     D)  $\frac{14}{17}$     E)  $\frac{1}{49}$

Ans: A

Difficulty: Hard

Section: 2.1

18. Let  $f(x) = \frac{1}{x^2}$ . Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[3, 5]$ .

Ans:  $-\frac{8}{225}$

Difficulty: Easy

Section: 2.1

19. Let  $f(x) = \frac{1}{x^2}$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = 5$ .

Ans:  $-\frac{2}{125}$

Difficulty: Easy

Section: 2.1

20. Let  $y = x^2 + 9$ . Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[-3, -2]$ .

Ans:  $-5$

Difficulty: Easy

Section: 2.1

21. Let  $y = x^2 + 12$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = -1$ .

Ans:  $-2$

Difficulty: Easy

Section: 2.1

22. Let  $y = \frac{1}{x-2}$ . Find the average rate of change of  $y$  with respect to  $x$  over the interval  $[4, 7]$ .

Ans:  $-\frac{1}{10}$

Difficulty: Medium

Section: 2.1

23. Let  $y = \frac{1}{x-3}$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = 7$ .

Ans:  $-\frac{1}{16}$

Difficulty: Medium

Section: 2.1

24. Let  $y = \frac{1}{x+4}$ . Find the average rate of change of  $y$  with respect to  $x$  over the given interval  $[3,4]$ .

Ans:  $-\frac{1}{56}$

Difficulty: Medium

Section: 2.1

25. Let  $y = \frac{4}{x+4}$ . Find the instantaneous rate of change of  $y$  with respect to  $x$  at the point  $x = 2$ .

Ans:  $-\frac{1}{9}$

Difficulty: Medium

Section: 2.1

26. Let  $f(x) = \frac{2}{4-x}$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent line at  $x_0 = 2$

Ans:  $\lim_{x_1 \rightarrow x_0} \frac{2}{(4-x_1)(4-x_0)} = \frac{2}{(4-x_0)^2}$

The slope of the tangent line at  $x_0 = 2$  is  $\frac{1}{2}$ .

Difficulty: Medium

Section: 2.1

27. Let  $f(x) = \frac{1}{x-2}$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent at  $x_0 = 5$ .

Ans:  $\lim_{x_1 \rightarrow x_0} \frac{-1}{(x_1-2)(x_0-2)} = \frac{-1}{(x_0-2)^2}$

The slope of the tangent line at  $x_0 = 5$  is  $-\frac{1}{9}$ .

Difficulty: Medium

Section: 2.1

28. Let  $f(x) = \frac{4}{x^6}$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent at  $x_0 = -3$ .

$$\text{Ans: } \lim_{x_1 \rightarrow -3} \frac{\frac{4}{x_1^6} - \frac{4}{-3^6}}{x_1 + 3} = \lim_{x_1 \rightarrow -3} \frac{4(-3^6 - x_1^6)}{729x_1^6(x_1 + 3)} = -\frac{24}{x_0^7}$$

The slope of the tangent line at  $x_0 = -3$  is  $-\frac{8}{729}$ .

Difficulty: Medium

Section: 2.1

29. Let  $f(x) = 3x^2$ . Find the slope of the tangent to the graph of  $f$  at a general point  $x_0$  using limits and find the slope of the tangent at  $x_0 = 3$ .

$$\text{Ans: } \lim_{x_1 \rightarrow x_0} 3(x_1 + x_0) = 6x_0$$

Slope of tangent at  $x_0 = 3$  is 18

Difficulty: Easy

Section: 2.1

30. A rock is dropped from a height of 400 feet and falls toward the earth in a straight line. In  $t$  seconds, the rock drops a distance of  $s = 16t^2$  feet. What is the average velocity of the rock while it is falling? Use limits to find the instantaneous velocity of the rock when it hits the ground.

Ans: Average velocity: 80 feet per second

Instantaneous velocity at ground = 160 feet per second

Difficulty: Medium

Section: 2.1

31. A particle moves in a straight line from its initial position so that after  $t$  seconds, its distance is given by  $s = t^2 + t$  feet from its initial position. Find the average velocity of the particle over the interval  $[3,5]$  seconds. Use limits to find the instantaneous velocity of the particle at  $t = 1$  second.

Ans: Average velocity = 9 feet per second

The instantaneous velocity at  $t = 1$  second is 3 feet per second.

Difficulty: Medium

Section: 2.1

32. A particle moves in a straight line from its initial position so that after  $t$  seconds, its

distance is given by  $s = \frac{t}{t+2}$  feet from its initial position. Find the average velocity of the particle over the interval  $[1,5]$  seconds. Use limits to find the instantaneous velocity of the particle at  $t = 1$  seconds.

Ans: Average velocity =  $\frac{2}{21}$  feet per second.

The instantaneous velocity at  $t = 1$  seconds is  $\frac{2}{9}$  feet per second.

Difficulty: Medium

Section: 2.1

33. Let  $f(x) = ax^2 + b$ , where  $a$  and  $b$  are constant. Use the method of Section 3.1 to show that the slope of the tangent to the graph of  $f$  at  $x = x_0$  is  $2ax_0$ .

$$m_{\tan} = \lim_{x_1 \rightarrow x_0} \frac{(ax_1^2 + b) - (ax_0^2 + b)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} \frac{a(x_1^2 - x_0^2)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} a(x_1 + x_0) = 2ax_0$$

Ans:

Difficulty: Hard

Section: 2.1

34. Let  $f(x) = ax^3 + b$ , where  $a$  and  $b$  are constants. Use the method of Section 3.1 to show

that the slope of the tangent to the graph of  $f$  at  $x = x_0$  is  $3ax_0^2$ .

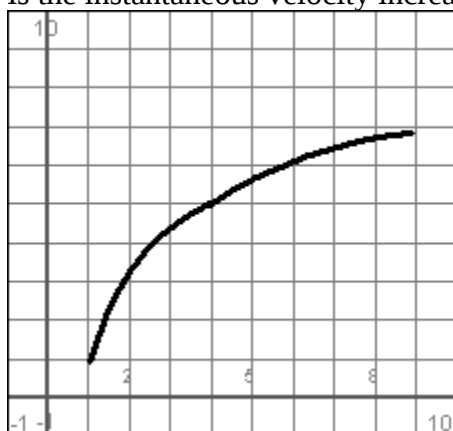
Ans:

$$m_{\tan} = \lim_{x_1 \rightarrow x_0} \frac{(ax_1^3 + b) - (ax_0^3 + b)}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} \frac{ax_1^3 - ax_0^3}{x_1 - x_0} = \lim_{x_1 \rightarrow x_0} a(x_1^2 + x_1x_0 + x_0^2) = 3ax_0^2$$

Difficulty: Medium

Section: 2.1

35. The graph shows the position versus time curve for a particle moving on a straight line. Is the instantaneous velocity increasing or decreasing with time?

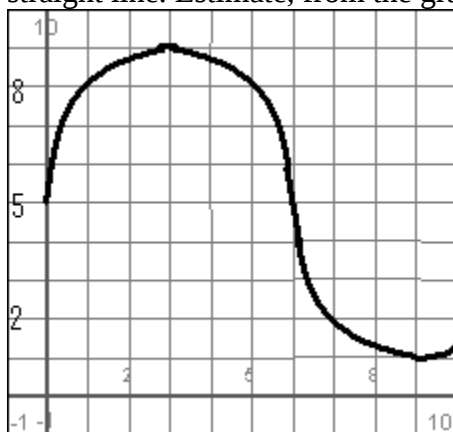


Ans: increasing

Difficulty: Easy

Section: 2.1

36. The figure shows the position versus time curve for a certain particle moving along a straight line. Estimate, from the graph, the instantaneous velocity at 8.



Ans: 1/3

Difficulty: Easy

Section: 2.1

37. Given  $f(x) = x^4 - 3$ , find the slope of the graph of  $f$  at the  $x$ -value  $x_0 = 3$ .

Ans: 108

Difficulty: Medium

Section: 2.1

38. Given  $f(x) = 11\sqrt[4]{x}$ , find the slope of the graph of  $f$  at  $x_0 = 16$ .

Ans:  $\frac{11}{32}$

Difficulty: Medium

Section: 2.1

39. Find the instantaneous rate of change of  $f(x) = \frac{11}{x^5}$  at  $x_0 = 2$ .

Ans:  $-\frac{55}{64}$

Difficulty: Medium

Section: 2.1

40. Find the instantaneous rate of change of  $f(x) = -2x^4 - 11$  at  $x_0 = 2$ .

Ans:  $-64$

Difficulty: Medium

Section: 2.1

41. Find the instantaneous rate of change of  $f(x) = -2x^2 - 3x + 11$  at  $x_0 = 4$ .

Ans:  $-19$

Difficulty: Medium

Section: 2.1

1. Find the equation of the tangent line to  $y = f(x) = 4x$  at  $x = 5$ .

- A)  $y = 4x - 5$    B)  $y = 4x$    C)  $y = -4x$    D)  $y = 4x + 5$    E)  $y = 4x - 20$

Ans: B

Difficulty: Easy

Section: 2.2

2. Find the equation of the tangent line to  $y = f(x) = \sqrt[3]{x} + 4$  at  $x = 8$ .

A)  $y = \frac{x}{12} + \frac{64}{4}$

D)  $y = \frac{x}{12} + \frac{4}{3}$

B)  $y = \frac{x}{12} + \frac{16}{3}$

E)  $y = \frac{x}{3} + \frac{64}{12}$

C)  $y = \frac{x}{3} + \frac{64}{3}$

Ans: B

Difficulty: Hard

Section: 2.2

3. Given  $y = 11\sqrt{x}$ , find  $\frac{dy}{dx}$ .

A)  $\frac{11\sqrt{x}}{x}$

B)  $\frac{\sqrt{x}}{2x}$

C)  $\frac{11}{2x}$

D)  $\frac{11\sqrt{x}}{2x}$

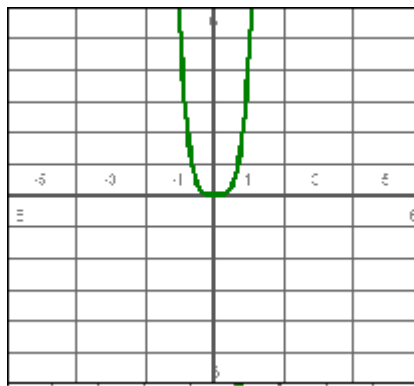
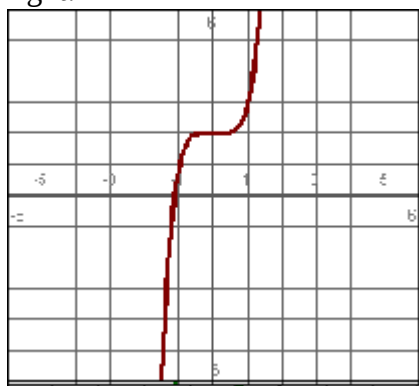
E)  $\frac{11\sqrt{x}}{2}$

Ans: D

Difficulty: Medium

Section: 2.2

4. Answer true or false. The derivative of the function graphed on the left is graphed on the right.



Ans: True

Difficulty: Medium

Section: 2.2

5. Answer true or false. Use a graphing utility to help in obtaining the graph of  $y = f(x) = |x - 11|$ . The derivative  $f'(x)$  is not defined at  $x = 11$ .

Ans: True

Difficulty: Medium

Section: 2.2

6. Find  $f'(t)$  if  $f(t) = 4t^6 - 8$ .

A)  $24t^5 - 8$     B)  $24t^6$     C)  $24t^6 - 8$     D)  $4t^5 - 8$     E)  $24t^5$

Ans: E

Difficulty: Easy

Section: 2.2

7.  $\lim_{h \rightarrow 0} \frac{(8+h)^2 - 64}{h}$  represents the derivative of  $f(x) = x^2$  at  $x = a$ . Find  $a$ .  
A) 64    B) 8    C) 1    D) 0    E) 56

Ans: B

Difficulty: Easy

Section: 2.2

8.  $\lim_{h \rightarrow 0} \frac{\sqrt[4]{81+h} - 3}{h}$  represents the derivative of  $f(x) = \sqrt[4]{x}$  at  $x = a$ . Find  $a$ .  
A) 3    B) 81    C) 78    D) 1    E) 0

Ans: B

Difficulty: Easy

Section: 2.2

9. Find an equation for the tangent line to the curve  $y = x^4 - 11$  at  $(2, 5)$ .

A)  $y = 32x$     D)  $y = 32x - 59$   
B)  $y = 32x - 11$     E)  $y = 32x^3 - 11$   
C)  $y = 32x^3 - 59$

Ans: D

Difficulty: Medium

Section: 2.2

10. Let  $f(x) = \sin x$ . Estimate  $f'\left(\frac{3\pi}{4}\right)$  by using a graphing utility.

A)  $\frac{\sqrt{2}}{2}$     B)  $-\frac{\sqrt{2}}{2}$     C)  $\frac{\sqrt{3}}{2}$     D)  $-\frac{1}{2}$     E)  $\frac{1}{2}$

Ans: B

Difficulty: Medium

Section: 2.2

11. An air source constantly increases the air supply rate of a balloon. The volume  $V$  in cubic feet is given by  $V(t) = 6t + 1$ ,  $[0 \leq t \leq 5]$ , where  $t$  is time in seconds. How fast is the volume of the balloon increasing at  $t = 5$ s?

A)  $30 \text{ ft}^3/\text{s}$    B)  $31 \text{ ft}^3/\text{s}$    C)  $1 \text{ ft}^3/\text{s}$    D)  $0 \text{ ft}^3/\text{s}$    E)  $6 \text{ ft}^3/\text{s}$

Ans: E

Difficulty: Medium

Section: 2.2

12. Answer true or false. Using a graphing utility it can be shown that  $f(x) = \sqrt[3]{|x-7|}$  is differentiable everywhere on  $[-10, 10]$ .

Ans: False

Difficulty: Easy

Section: 2.2

13. Answer true or false. A graphing utility can be used to determine that

$f(x) = \begin{cases} x^8 + 8, & x \leq 0 \\ x^2 + 8, & x > 0 \end{cases}$  is differentiable at  $x = 0$ .

Ans: True

Difficulty: Medium

Section: 2.2

14. Answer true or false. A graphing utility will show that differentiable at  $x = -1$ .

$$f(x) = \begin{cases} 3x^6 - 12, & x \leq -1 \\ 3x^{12} - 12, & x > -1 \end{cases}$$
 is

Ans: False

Difficulty: Medium

Section: 2.2

15. Find  $f'(x)$  if  $f(x) = 5x^4$ .

A)  $20x^4$    B)  $20x^3$    C)  $5x^3$    D)  $4x^3$    E)  $\frac{5x^3}{4}$

Ans: B

Difficulty: Easy

Section: 2.2

16. Find  $f'(x)$  if  $f(x) = 2x^3 - 3x$ .

A)  $6x^2 - 3$    B)  $6x^2$    C)  $6x^3 - 3$    D)  $2x^2 - 3$    E)  $6x^3$

Ans: A

Difficulty: Easy

Section: 2.2

17. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = 2x^2 - 5x$  and find the equation of the tangent to the graph of  $f$  at  $x = 2$ .

Ans: Derivative:  $f'(x) = 4x - 5$

Tangent line:  $y = 3x - 8$

Difficulty: Medium

Section: 2.2

18. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = 5x^2 - 8x + 6$ .

Ans:  $10x - 8$

Difficulty: Easy

Section: 2.2

19. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = 4x^4 + 7$  and find the equations of the tangent line and the normal line to the graph of  $f$  at  $x = -4$ .

Ans: Derivative:  $y = 16x^3$

Tangent line:  $y = -1,024x - 3,065$

Normal line:  $y = \frac{x}{1,024} + \frac{263,937}{256}$

Difficulty: Medium

Section: 2.2

20. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = 2x^2 - 2x$  and find the equation of the tangent line and the normal line to the graph of  $f$  at  $x = 2$ .

Ans:  $f'(x) = 4x - 2$

Tangent line:  $y = 6x - 8$

Normal line:  $y = -\frac{1}{6}x + \frac{11}{3}$

Difficulty: Medium

Section: 2.2

21. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = \sqrt{2x}$  and find the equations of the tangent line and the normal line to the graph of  $f$  at  $x = 2$ .

Ans:  $f'(x) = \frac{\sqrt{2}}{2\sqrt{x}}$

Tangent:  $y = \frac{1}{2}x + 1$

Normal:  $y = -2x + 6$

Difficulty: Medium

Section: 2.2

22. Let  $y = \sqrt{6x+6}$ . Use the definition of the derivative to find  $\frac{dy}{dx}$ .

Ans:  $\frac{dy}{dx} = \frac{3}{\sqrt{6x+6}}$

Difficulty: Medium

Section: 2.2

23. Let  $y = \frac{-3}{8x+7}$ . Use the definition of the derivative to find  $\frac{dy}{dx}$ .

Ans:  $\frac{dy}{dx} = \frac{24}{(8x+7)^2}$

Ans:

Difficulty: Medium

Section: 2.2

24. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = \frac{1}{3-x}$ .

Ans:  $f'(x) = \frac{1}{(3-x)^2}$

Ans:

Difficulty: Medium

Section: 2.2

25. Given that  $f(2) = -2$  and  $f'(2) = -2$ , find an equation for the tangent line to the graph of  $y = f(x)$  at the point where  $x = 2$ .

Ans:  $y = -2x + 2$

Difficulty: Hard

Section: 2.2

26. Given that  $f(1) = 5$  and  $f'(1) = 6$ , find an equation for the tangent line to the graph of  $y = f(x)$  at the point where  $x = 1$ .

Ans:  $y = 6x - 1$

Difficulty: Hard

Section: 2.2

27. Use the definition of the derivative to calculate  $f'(x)$  if  $f(x) = \frac{2}{\sqrt{3x}}$  and find the equations of the tangent line and the normal line to the graph of  $f$  at  $x = 12$ .

$$f'(x) = \frac{-3}{\sqrt{(3x)^3}}$$

Ans:

$$\text{Tangent line: } y = -\frac{1}{72}x + \frac{1}{2}$$

$$\text{Normal line: } y = 72x + \frac{2,593}{3}$$

Difficulty: Hard

Section: 2.2

28. The volume of a sphere is given by  $\frac{4}{3}\pi r^3$  where  $r$  is the radius of the sphere. Use the method of Section 3.2 to find the instantaneous rate of change of  $V$  with respect to  $r$  when  $r = 6$ .

Ans: The instantaneous rate of change =  $144\pi$

Difficulty: Easy

Section: 2.2

29. The surface area of a sphere is given by  $S = 4\pi r^2$  where  $r$  is the radius of the sphere. Use the method of Section 3.2 to find the instantaneous rate of change of  $S$  with respect to  $r$  when  $r = 2$ .

Ans: The instantaneous rate of change of  $S$  with respect to  $r$  at  $r = 2$  is  **$16\pi$** .

Difficulty: Easy

Section: 2.2

30. The volume of a sphere is given by  $V = \frac{\pi}{6}D^3$  where  $D$  is the diameter of the sphere. Use the method of Section 3.2 to find the instantaneous rate of change of  $V$  with respect to  $D$  when  $D = 2$ .

Ans: The instantaneous rate at which  $V$  changes with respect to  $D$  when  $D = 2$  is  **$2\pi$** .

Difficulty: Easy

Section: 2.2

31. Show that  $f(x) = \begin{cases} x^2 - 5, & x \leq 1 \\ x - 5, & x > 1 \end{cases}$  is continuous but not differentiable at  $x = 1$ .

Ans: Continuous:  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 - 5) = -4$  and  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (x - 5) = -4$ , so it is continuous at  $x = 1$ .

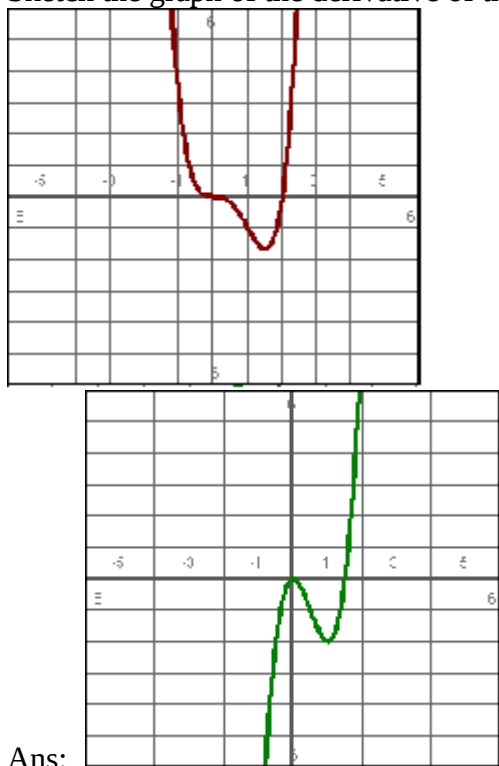
Differentiable:  $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{(1+h) - 1} = \lim_{h \rightarrow 0^-} \frac{2h + h^2}{h} = \lim_{h \rightarrow 0^-} (2 + h) = 2$   
 $\neq \lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{(1+h) - 1} = \lim_{h \rightarrow 0^+} \frac{h}{h} = 1$

, so it is NOT differentiable at  $x = 1$ .

Difficulty: Hard

Section: 2.2

32. Sketch the graph of the derivative of the function whose graph is shown.



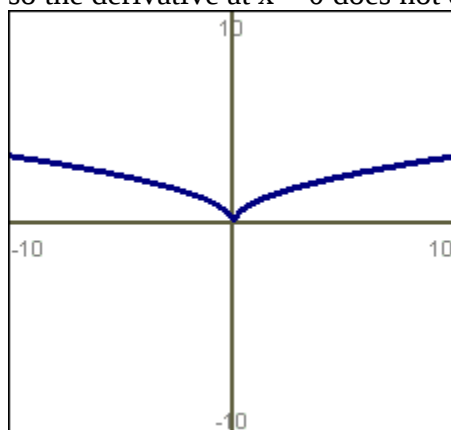
Ans:

Difficulty: Medium

Section: 2.2

33. Use a graphing utility to show that  $y = \sqrt[4]{x^2}$  does not have a derivative at  $x = 0$ .

Ans: To the left of 0, the tangent is negative and to the right of 0, the tangent is positive, so the derivative at  $x = 0$  does not exist.

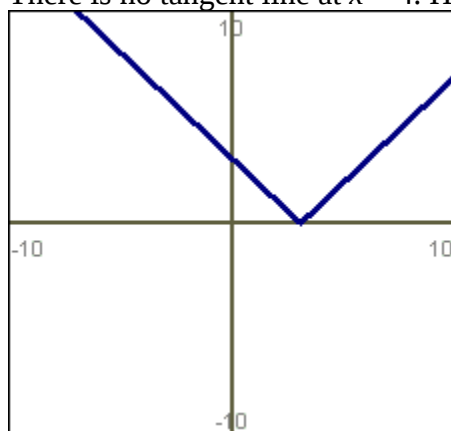


Difficulty: Easy

Section: 2.2

34. Use a graphing utility to show that  $y = |x - 4|$  does not have a derivative everywhere.

Ans: There is no tangent line at  $x = 4$ . Hence, there is no derivative at  $x = 4$ .



Difficulty: Easy

Section: 2.2

35. Let  $f(x) = 2^x$ . Estimate  $f'(2)$  using a graphing utility.

Ans: 2.773

Difficulty: Hard

Section: 2.2

36. Let  $f(x) = \cos x$ . Estimate  $f'(1)$  using a graphing utility.

Ans: -0.841

Difficulty: Easy

Section: 2.2

37. The cost of manufacturing an object is given by  $C = f(x)$  dollars. Estimate the cost of making the 310<sup>th</sup> item if  $f'(309) = \$74$ .

Ans: \$74

Difficulty: Easy

Section: 2.2

1. Find  $dy/dx$  if  $y = 5x^8$ .

- A)  $40x^8$    B)  $5x^7$    C)  $40x^7$    D)  $40x$    E)  $x^7$

Ans: C

Difficulty: Easy

Section: 2.3

2. Find  $dy/dx$  if  $y = \sqrt{2\pi}$ .

- A)  $\frac{2\sqrt{\pi}}{\pi}$    B)  $-\frac{\sqrt{2\pi}}{4\pi}$    C) 0   D) 1   E)  $\frac{\sqrt{2\pi}}{2\pi}$

Ans: C

Difficulty: Easy

Section: 2.3

3. Find  $dy/dx$  if  $y = 3(x^6 - 6x + 1)$ .

- A)  $3x^5 - 6x - 18$    B)  $6x^5 - 18$    C)  $18x^5 - 18$    D)  $18x^5 - 6$    E)  $3x^5 - 6$

Ans: C

Difficulty: Easy

Section: 2.3

4. Answer true or false. If  $f(x) = \sqrt[4]{x} + 7x$ ,  $f'(x) = \frac{\sqrt[4]{x}}{4} + 7$ .

Ans: False

Difficulty: Medium

Section: 2.3

5.  $y = 4x^2 + 9x + 2$ . Find  $d^2y/dx^2$ .

- A) 8   B)  $8x + 9$    C) 4   D)  $4x + 2$    E) 2

Ans: A

Difficulty: Medium

Section: 2.3

6.  $y = 2x^{-3} + 2x$ . Find  $y'''$ .

- A) 12   B) -120   C)  $-120x^{-6}$    D)  $-120x^{-6} + 2$    E)  $12x^{-6} + 2$

Ans: C

Difficulty: Hard

Section: 2.3

7. Answer true or false.  $y = y''' + 5y' - 2$  is satisfied by  $y = x$ .

Ans: False

Difficulty: Hard

Section: 2.3

8. Use a graphing utility to locate all horizontal tangent lines to the curve  $y = x^3 + 3x^2 + 12$ .  
 A)  $x = 0$     B)  $x = -3$  and  $x = 0$     C)  $x = -2$  and  $x = 0$     D)  $x = -2$     E)  $x = -3$

Ans: C

Difficulty: Medium

Section: 2.3

9. Find the  $x$ -coordinate of the point on the graph of  $y = x^4 + 4$  where the tangent line is parallel to the secant line that cuts the curve at  $x = 2$  and at  $x = 3$ .

A)  $\sqrt[3]{\frac{65}{4}}$     B) 85    C) 20    D)  $\sqrt[4]{\frac{65}{4}}$     E)  $\frac{65}{4}$

Ans: A

Difficulty: Hard

Section: 2.3

10. The position of a moving particle is given by  $s(t) = 3t^5 - 1$  where  $t$  is time in seconds. The velocity in m/s is given by  $ds/dt$ . Find the velocity at  $t = 3$ .

A) 3,645 m/s    B) 243 m/s    C) 405 m/s    D) 1,215 m/s    E) 1,214 m/s

Ans: D

Difficulty: Medium

Section: 2.3

11. Find  $\frac{dy}{dx}$ , if  $y = \frac{5x^6 - 6}{6}$ .

A)  $30x^5$     B)  $6x^5 - 6$     C)  $\frac{30x^5 - 6}{6}$     D)  $5x^5$     E)  $\frac{30x^5}{6} + 6$

Ans: D

Difficulty: Easy

Section: 2.3

12. Find  $\frac{dy}{dx}$  if  $y = \frac{5x^4 + 8x^2 + \sqrt{x}}{x}$ .

$\frac{dy}{dx} = 15x^2 + 8 - \frac{1}{2\sqrt{x^3}}$

Ans:

Difficulty: Easy

Section: 2.3

13. Find  $\frac{dy}{dx}$  if  $y = \frac{x^2 + 5x}{7 - 4x}$ .

$$\frac{dy}{dx} = \frac{-4x^2 + 14x + 35}{(7 - 4x)^2}$$

Ans:

Difficulty: Medium

Section: 2.3

14. Find  $f''(2)$  if  $f(x) = \frac{-32}{x^2} + \frac{x^5}{5}$ .

$$f'(x) = \frac{64}{x^3} + x^4$$

$$f''(x) = \frac{-192}{x^4} + 4x^3$$

$$\text{So: } f''(2) = 20.$$

Difficulty: Hard

Section: 2.3

15. Find  $\frac{d^2y}{dt^2}$  if  $y = -\frac{2}{t} - \frac{7}{t^2}$ .

$$f'(t) = \frac{2}{t^2} + \frac{14}{t^3}$$

Ans:

$$f''(t) = -\frac{4}{t^3} - \frac{42}{t^4}$$

Difficulty: Hard

Section: 2.3

16. Find  $\frac{dv}{dh}$  if  $v = 2\pi \left( ah^3 - \frac{1}{4}h^4 \right)$  and **a** is a constant.

$$\text{Ans: } \frac{dv}{dh} = 2\pi (3ah^2 - h^3)$$

Difficulty: Easy

Section: 2.3

17. Find equations for the tangents to the graph of  $y = 35 - 2x - x^2$  at those points where the curve intersects the  $x$ -axis.

Ans:  $y' = -2 - 2x$

The tangent lines are:

At  $x = -7$ , the tangent line is:  $y = 12x + 84$ .

At  $x = 5$ , the tangent line is:  $y = -12x + 60$ .

Difficulty: Hard

Section: 2.3

18. Find equations for the tangents and normals to the graph of  $y = 30 - x - x^2$  at the points where the curve intersects the  $x$ -axis.

Ans:  $y' = -1 - 2x$

So the tangent lines are:

At  $x = 5$ , the equation of the tangent line is:  $y = -11x + 55$ .

At  $x = -6$ , the equation of the tangent line is:  $y = 11x + 66$ .

The normal lines are:

At  $x = 5$ , the equation of the normal line is:  $y = \frac{1}{11}x - \frac{5}{11}$ .

At  $x = -6$ , the equation of the normal line is:  $y = -\frac{1}{11}x - \frac{6}{11}$ .

Difficulty: Hard

Section: 2.3

19. Find the points on the graph of  $y = 2x^3 + 21x^2 + 60x + 30$  at which the tangent is parallel to the  $x$ -axis.

Ans: Equations of tangent that are parallel to the  $x$ -axis:

at  $x = -5$ , the equation of the tangent is:  $y = 5$

at  $x = -2$ , the equation of the tangent is:  $y = -22$

Difficulty: Medium

Section: 2.3

20. Show that the parabola  $y = -x^2$  and the line  $x - 10y - 255 = 0$  intersect at right angles at one of their points of intersection.

Ans: For the line, the slope is described by  $1 - 10 \frac{dy}{dx} = 0$  or  $\frac{dy}{dx} = \frac{1}{10}$ .

For the parabola, the slope is described by  $\frac{dy}{dx} = -2x$ .

At  $x = 5$ , the slope of the line is  $\frac{1}{10}$ .

And the slope of the parabola is  $-10$ .

So the (tangent line to the) parabola is perpendicular to the line at  $x = 5$ .

Difficulty: Hard

Section: 2.3

21. Show that the parabola  $y = x^2$  and the line  $x + 6y - 57 = 0$  intersect at right angles at one of their points of intersection.

Ans: For the line, the slope is described by  $1 + 6 \frac{dy}{dx} = 0$  or  $\frac{dy}{dx} = -\frac{1}{6}$ .

For the parabola, the slope is described by  $\frac{dy}{dx} = 2x$ .

At  $x = 3$ , the slope of the line is  $-\frac{1}{6}$ .

And the slope of the parabola is  $6$ .

So the (tangent line to the) parabola is perpendicular to the line at  $x = 3$ .

Difficulty: Hard

Section: 2.3

22. Find  $\frac{dy}{dx}$  if  $y = -8x^{10}$ .

Ans:  $-80x^9$

Difficulty: Easy

Section: 2.3

23. Find  $\frac{dy}{dx}$  if  $y = \alpha^{11}$ . (Alpha is a constant.)

Ans: 0

Difficulty: Easy

Section: 2.3

24. Find  $\frac{dy}{dx}$  if  $y = 3\sqrt{x} - 9x$ .

Ans:  $\frac{3}{2\sqrt{x}} - 9$  or  $\frac{3\sqrt{x}}{2x} - 9$

Difficulty: Medium

Section: 2.3

25. If  $y = x^8 + 4x^3$ , find  $\frac{d^2y}{dx^2}$ .

Ans:  $\frac{dy}{dx} = 8x^7 + 12x^2$

$\frac{d^2y}{dx^2} = 56x^6 + 24x$

Difficulty: Medium

Section: 2.3

26. If  $y = x^4 - 4x + 5$ , find  $\frac{d^2y}{dx^2}$ .

Ans:  $\frac{dy}{dx} = 4x^3 - 4$ ,

$\frac{d^2y}{dx^2} = 12x^2$

Difficulty: Easy

Section: 2.3

27. Find  $y'''(3)$  if  $y = x^5$ .

Ans: 540

Difficulty: Easy

Section: 2.3

28. Find  $f'(s)$  if  $f(s) = (6s^2 + 8)(s^2 - 7s)$ .

Ans:  $f'(s) = 12s(s^2 - 7s) + (2s - 7)(6s^2 + 8) = 24s^3 - 126s^2 + 16s - 56$

Difficulty: Medium

Section: 2.3

1. Answer true or false. If  $y = \frac{1}{8x-2}$ ,  $y' = \frac{1}{8}$ .

Ans: False

Difficulty: Easy

Section: 2.4

2. If  $y = \frac{6x}{x-12}$ , find  $\frac{dy}{dx}\bigg|_3$ .

A)  $\frac{8}{9}$     B)  $-\frac{8}{9}$     C)  $-\frac{6}{9}$     D)  $\frac{6}{9}$     E)  $-\frac{8}{225}$

Ans: B

Difficulty: Medium

Section: 2.4

3. If  $y = \frac{6}{x+7}$ ,  $y'(2) =$

A) 0    B)  $\frac{2}{27}$     C)  $-\frac{14}{27}$     D)  $-\frac{2}{27}$     E)  $-\frac{7}{27}$

Ans: D

Difficulty: Medium

Section: 2.4

4.  $g(x) = x^4 f(x)$ . Find  $g'(4)$ , given that  $f(4) = 3$  and  $f'(4) = 6$ .

A) 1,792    B) 1,024    C) 2,304    D) 4,608    E) 1,536

Ans: C

Difficulty: Hard

Section: 2.4

5. Answer true or false. If  $f$ ,  $g$ , and  $h$  are differentiable functions, and  $h \neq 0$  anywhere on its

domain, then  $\left(\frac{fg}{h}\right)' = \frac{fhg' + ghf' - fgh'}{h^2}$ .

Ans: True

Difficulty: Medium

Section: 2.4

6. If  $y = \frac{1}{\sqrt{x}-4}$ , find  $y'(25)$ .

- A)  $-\frac{1}{4}$     B)  $\frac{1}{4}$     C)  $\frac{1}{10}$     D)  $-\frac{1}{25}$     E)  $-\frac{1}{10}$

Ans: E

Difficulty: Hard

Section: 2.4

7. Find  $\frac{dy}{dx}$  if  $y = 3(x^2 - 2x)(6 + x^5)$ .

$$\frac{dy}{dx} = 3(2x - 2)(6 + x^5) + 15x^4(x^2 - 2x) = 21x^6 - 36x^5 + 36x - 36$$

Ans:

Difficulty: Medium

Section: 2.4

8. Find  $f'(x)$  if  $f(x) = \frac{6x^2 + 6}{3x^2 + 8x}$ .

$$f'(x) = \frac{12x(3x^2 + 8x) - (6x + 6)(6x^2 + 6)}{(3x^2 + 8x)^2} = \frac{48x^2 - 36x - 48}{(3x^2 + 8x)^2}$$

Ans:

Difficulty: Medium

Section: 2.4

9. If  $f(3) = 4$ ,  $f'(3) = -5$ ,  $g(3) = 2$ ,  $g'(3) = -4$ , find  $F'(3)$  if  $F(x) = 2f(x) - g(x)$ .

Ans: -6

Difficulty: Easy

Section: 2.4

10. If  $f(4) = 7$ ,  $f'(4) = 2$ ,  $g(4) = 2$ ,  $g'(4) = -2$ , find  $F'(4)$  if  $F(x) = 6f(x)g(x)$ .

Ans: -60

Difficulty: Easy

Section: 2.4

11. If  $f(6) = 48$ ,  $f'(6) = 5$ ,  $g(6) = 4$ ,  $g'(6) = -3$ , find  $F'(6)$  if  $F(x) = \frac{4f(x)}{4g(x)}$ .

Ans:  $\frac{41}{4}$  of  $10\frac{1}{4}$

Difficulty: Easy

Section: 2.4

12. Find  $f'(u)$  if  $f(u) = \frac{u^2 - 7}{4u^2 - 4}$ .

$$f'(u) = \frac{2u(4u^2 - 4) - 8u(u^2 - 7)}{(4u^2 - 4)^2} = \frac{48u}{(4u^2 - 4)^2}$$

Ans:

Difficulty: Medium

Section: 2.4

13. Find  $f'(t)$  if  $f(t) = \frac{3t^3 - 12t^2}{3t^4}$ .

Ans:  $y = t^{-1} - 4t^{-2}$  So  $y' = -t^{-2} + 8t^{-3}$ .

Difficulty: Medium

Section: 2.4

14. Find  $\frac{dy}{dx}$  if  $y = (x^2 - 1)(x^3 + 3x)$ .

Ans:  $\frac{dy}{dx} = 2x(x^3 + 3x) + (3x^2 + 3)(x^2 - 1) = 5x^4 + 6x^2 - 3$

Difficulty: Medium

Section: 2.4

15. Find  $f'(x)$  if  $f(x) = (x^2 + 5)(x^3 - 4x^2 + x)$ .

Ans:  $f'(x) = 5x^4 - 16x^3 + 18x^2 - 40x + 5$

Difficulty: Medium

Section: 2.4

16. Find the equations of the tangent and normal to the graph of  $y = \frac{x+1}{x-1}$  at  $x = 4$ .

Ans: Tangent line:  $y = -\frac{2}{9}x + \frac{23}{9}$

Normal line:  $y = \frac{9}{2}x - \frac{49}{3}$

Difficulty: Hard

Section: 2.4

1. Find  $f'(x)$  if  $f(x) = x^4 \cos x$ .

A)  $4x^3 \cos x - x^4 \sin x$

B)  $4x^3 \cos x$

C)  $4x^3 \sin x$

D)  $-4x^3 \cos x$

E)  $-4x^3 \sin x$

Ans: A

Difficulty: Easy

Section: 2.5

2. Find  $f'(x)$  if  $f(x) = \cos x \tan x$ .

A)  $\sec x$     B)  $-\cos x$     C)  $-\sin x \tan x + 1$     D)  $\cos x$     E)  $-\tan x$

Ans: D

Difficulty: Medium

Section: 2.5

3. Find  $f'(x)$  if  $f(x) = \csc^2 x + \cot^2 x$ .

A)  $2 \csc x + 2 \cot x$     B)  $2 \csc x - 2 \cot x$     C)  $-1$     D)  $1$     E)  $0$

Ans: E

Difficulty: Easy

Section: 2.5

4. Find  $d^2y/dx^2$  if  $y = x \tan x$ .

A)  $2x \tan x \sec x$

B)  $x \sec x$

C)  $2 \sec^2 x + 2x \tan x \sec^2 x$

D)  $\sec x$

E)  $-2x \tan x \sec x$

Ans: C

Difficulty: Medium

Section: 2.5

5. Answer true or false. If  $y = \sin x$ ,  $d^2y/dx^2 = \sin x$ .

Ans: False

Difficulty: Medium

Section: 2.5

6. Find the equation of the line tangent to the graph of  $y = \sin x$  at the point where  $x = 0$ .

A)  $y = \cos x$     B)  $y = x$     C)  $y = -x$     D)  $y = x$     E)  $y = x$

Ans: E

Difficulty: Medium

Section: 2.5

7. Find the  $x$ -coordinates of all points in the interval  $[-2\pi, 2\pi]$  at which the graph of  $f(x) = \csc 2x$  has a horizontal tangent line.

- A)  $\left(\pm\frac{\pi}{4}, \pm\frac{\pi}{2}\right)$  D)  $\left(\pm\frac{\pi}{4}, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \pm\frac{3\pi}{2}\right)$   
 B)  $\left(\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}\right)$  E)  $\left(\pm\pi, \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \pm 2\pi\right)$   
 C)  $\left(\pm\frac{3\pi}{4}, \pm\frac{3\pi}{2}\right)$

Ans: B

Difficulty: Easy

Section: 2.5

8. Find  $d^{75}$  of  $\cos x/dx^{75}$ .

- A)  $-\cos x$  B)  $-\sin x$  C)  $\cos x$  D) 0 E)  $\sin x$

Ans: E

Difficulty: Medium

Section: 2.5

9. Find all  $x$ -values on  $[0, 2\pi]$  where  $f(x)$  is not differentiable.  $f(x) = \cos x \cot x$

- A)  $\frac{\pi}{2}, \frac{3\pi}{2}$  B)  $0, \pi, 2\pi$  C)  $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$  D) 0 E) none

Ans: B

Difficulty: Medium

Section: 2.5

10. Answer true or false. If  $x$  is given in radians, the derivative formula for  $y = \sin x$  in

degrees is  $y' = \frac{\pi}{180} \cos x$ .

Ans: False

Difficulty: Easy

Section: 2.5

11. A rock at an elevation angle of  $\theta$  is falling in a straight line. If at a given instant it has an angle of elevation of  $\theta = \frac{\pi}{3}$  and is a horizontal distance  $s$  from an observer, find the rate at which the rock is falling with respect to  $\theta$ .

- A)  $\frac{\sec^2\left(\frac{\pi}{3}\right)}{s}$                       D)  $\sec^2\left(\frac{s\pi}{3}\right)$   
 B)  $s \times \sec^2\left(\frac{\pi}{3}\right)$                       E)  $s \times \sec\left(\frac{\pi}{3}\right)$   
 C)  $\sec^2\left(\frac{\pi}{3}\right)$

Ans: B

Difficulty: Medium

Section: 2.5

12. Answer true or false. If  $f(x) = \cos x \sec x - \sin x$ ,  $f'(x) = 1 - \cos x$ .

Ans: False

Difficulty: Medium

Section: 2.5

13. Answer true or false. If  $f(x) = \frac{1}{\sin x}$ ,  $f'(x) = \frac{1}{\cos x}$ .

Ans: False

Difficulty: Easy

Section: 2.5

14. Answer true or false.  $f(x) = \frac{\csc x}{1 - \csc x}$  is differentiable everywhere.

Ans: False

Difficulty: Easy

Section: 2.5

15. If  $y = x^6 \cos x$ , find  $d^2y/dx^2$ .

- A)  $30x^4 \cos x - 12x^5 \sin x + x^6 \cos x$                       D)  $30x^4 \cos x - 12x^5 \sin x$   
 B)  $30x^4 \cos x - 6x^5 \sin x - x^6 \cos x$                       E)  $6x^5 \cos x - x^6 \sin x$   
 C)  $30x^4 \cos x - 12x^5 \sin x - x^6 \cos x$

Ans: C

Difficulty: Hard

Section: 2.5

16. If  $y = x^7 \cos x$ , find  $dy/dx$ .

A)  $7x^6 \cos x + x^7 \sin x$

B)  $7x^6 \sin x - x^7 \cos x$

C)  $42x^5 \cos x - 14x^6 \sin x - x^7 \cos x$

D)  $42x^5 \cos x - 14x^6 \sin x$

E)  $7x^6 \cos x - x^7 \sin x$

Ans: E

Difficulty: Hard

Section: 2.5

17. Find  $d^6y/dx^6$  if  $y = \cos x$ .

A)  $-\cos x$

B)  $\cos x$

C)  $\sin x$

D)  $-\sin x$

E)  $\sin x \cos x$

Ans: A

Difficulty: Easy

Section: 2.5

18. Find  $f'(x)$  if  $f(x) = x \tan x$ .

Ans:  $x \sec^2 x + \tan x$

Difficulty: Medium

Section: 2.5

19. Find  $f''(x)$  if  $f(x) = x \sin x$ .

Ans:  $-x \sin x + 2 \cos x$

Difficulty: Medium

Section: 2.5

20. Find  $\frac{dy}{dx}$  if  $y = \frac{\cos x}{x^4}$ .

$\frac{dy}{dx} = \frac{-x \sin x - 4 \cos x}{x^5}$

Ans:

Difficulty: Medium

Section: 2.5

21. Find  $\frac{dy}{dx}$  if  $y = \sec x \tan x$ .

Ans:  $\sec x \tan^2 x + \sec^3 x$

Difficulty: Hard

Section: 2.5

22. Find  $f'(x)$  if  $f(x) = \frac{\cot x}{1 + \csc x}$ .

$$f'(x) = \frac{-\csc x}{(1 + \csc x)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

23. Find  $f'(x)$  if  $f(x) = (7x^4 + 9) \cos x$ .

Ans:  $28x^3 \cos x - (7x^4 + 9) \sin x$

Difficulty: Medium

Section: 2.5

24. Differentiate  $y = \frac{\csc x}{\sqrt{x}}$ .

$$\frac{dy}{dx} = \frac{-2x \csc x \cot x - \csc x}{2\sqrt{x^3}}$$

Ans:

Difficulty: Hard

Section: 2.5

25. Find  $\frac{dy}{dx}$  if  $y = \frac{\sin x}{1 - \cos x}$ .

$$\frac{dy}{dx} = \frac{-1}{1 - \cos x}$$

Ans:

Difficulty: Medium

Section: 2.5

26. Find  $\frac{dy}{dx}$  if  $y = (3x^2 + 5x) \cot x$ .

Ans:  $\frac{dy}{dx} = (6x + 5) \cot x - (3x^2 + 5x) \csc^2 x$

Difficulty: Medium

Section: 2.5

27. Find  $y''(x)$  if  $y = 8 \cos x + 5 \sin x + \frac{x^6}{6}$ .

$$\frac{dy}{dx} = -8 \cos x - 5 \sin x + 5x^4$$

Ans:

Difficulty: Hard

Section: 2.5

28. Find  $f'(\theta)$  if  $f(\theta) = \frac{1}{1-5\sin\theta}$  .

$$f'(\theta) = \frac{5\cos\theta}{(1-5\sin\theta)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

29. Find  $\frac{dy}{dx}$  if  $y = 8x \sin x - 3 \cos x + x^2 \cos x$ .

$$\frac{dy}{dx} = 11\sin x - x^2 \sin x + 10x \cos x$$

Ans:

Difficulty: Hard

Section: 2.5

30. Find  $f'(\theta)$  if  $f(\theta) = \frac{1+\sin\theta}{1-\sin\theta}$  .

$$f'(\theta) = \frac{2\cos\theta}{(1-\sin\theta)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

31. Find  $\frac{dy}{dt}$  if  $y = \frac{1+\tan t}{1-\tan t}$  .

$$\frac{dy}{dt} = \frac{2\sec^2 t}{(1-\tan t)^2}$$

Ans:

Difficulty: Medium

Section: 2.5

32. Show by use of a trigonometric identity that  $\frac{d}{dx} |\tan x - x| = \tan^2 x$  .

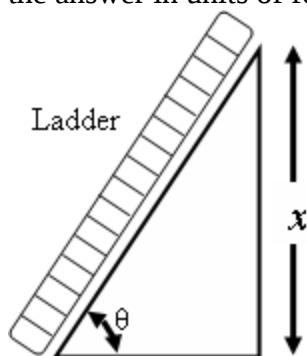
$$\frac{d}{dx} |\tan x - x| = \sec^2 x - 1 = \tan^2 x$$

Ans:

Difficulty: Easy

Section: 2.5

33. A 16 foot long ladder leans against a wall at an angle  $\theta$  with the horizontal as shown in the figure. The top of the ladder is  $x$  feet above the ground. If the bottom of the ladder is pushed toward the wall, find the rate at which  $x$  changes with  $\theta$  when  $\theta = 45^\circ$ . Express the answer in units of feet/degree.



Ans:  $\sin \theta = \frac{x}{16}$   
 $x = 16 \sin \theta$   
 $\frac{dx}{d\theta} = 16 \cos \theta$

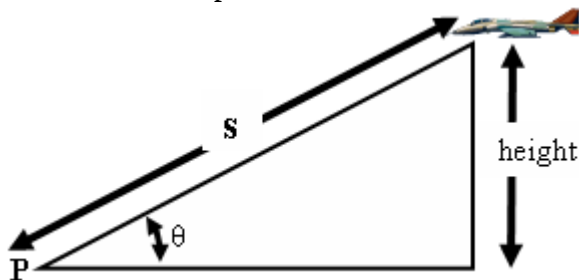
$$\left. \frac{dx}{d\theta} \right|_{\theta=45^\circ} = 16 \cos 45^\circ = 16 \times 0.7071 = 11.3137 \text{ ft / degree}$$

Since  $\theta = 45^\circ$

Difficulty: Medium

Section: 2.5

34. An airplane is flying on a horizontal path at a height of 6000ft, as shown in the figure. At what rate is the distance  $s$  between the airplane and the fixed point  $P$  changing with  $\theta$  when  $\theta = 20^\circ$ . Express the answer in units of feet/degree.



$$\begin{aligned} \text{Ans: } \csc \theta &= \frac{s}{6000} \\ s &= 6000 \csc \theta \\ \frac{ds}{d\theta} &= -6000 \csc \theta \cot \theta \\ \text{Since } \theta &= 20^\circ \quad \left. \frac{ds}{d\theta} \right|_{\theta=20^\circ} = \end{aligned}$$

$$6000 \csc 20^\circ \cot 20^\circ = 48,198.519 \text{ ft/degree}$$

Difficulty: Medium

Section: 2.5

35.  $f(x) = 4 \cos x + 7 \sin x$ . Find  $f'(x)$ .

$$\text{Ans: } f'(x) = -4 \sin x + 7 \cos x$$

Difficulty: Medium

Section: 2.5

36.  $f(x) = x^2 \cos x + \sin x$ . Find  $f'(x)$ .

$$\text{Ans: } -x^2 \sin x + 2x \cos x + \cos x$$

Difficulty: Medium

Section: 2.5

37.  $f(x) = 3x \cot x$ . Find  $f'(x)$ .

$$\text{Ans: } 3 \cot x - 3x \csc^2 x$$

Difficulty: Medium

Section: 2.5

38.  $f(x) = \frac{\csc x}{x^5}$ . Find  $f'(x)$ .

$$f'(x) = \frac{-x \csc x \cot x - 5 \csc x}{x^6}$$

Ans:

Difficulty: Medium

Section: 2.5

39. Find  $\frac{dy}{dx}$  if  $y = \tan x$ .

Ans:  $\sec^2 x$

Difficulty: Easy

Section: 2.5

1.  $f(x) = \sqrt{x^5 + 3}$ .  $f'(x) =$

A)  $\frac{5x^4}{\sqrt{x^5 + 3}}$     B)  $\frac{5x^4 + 3}{2\sqrt{x^5 + 3}}$     C)  $\frac{5x^4}{2}$     D)  $\frac{5x^4}{2\sqrt{x^5 + 3}}$     E)  $\frac{5x^5}{2}$

Ans: D

Difficulty: Medium

Section: 2.6

2.  $f(x) = (x^4 - 3)^{38}$ .  $f'(x) =$

A)  $38x^3 (x^4 - 3)^{37}$     D)  $152(x^4 - 3)^{37}$   
 B)  $4x^3 (x^4 - 3)^{37}$     E)  $38(x^4 - 3)^{37}$   
 C)  $152x^3 (x^4 - 3)^{37}$

Ans: C

Difficulty: Medium

Section: 2.6

3.  $f(x) = \sin(6x)$ .  $f'(x) =$

A)  $6 \cos(6x)$     B)  $\cos(6x)$     C)  $6 \cos(x)$     D)  $6 \sin(6x)$     E)  $\sin(6x)$

Ans: A

Difficulty: Medium

Section: 2.6

4. Answer true or false. If  $f(x) = \sqrt{\cos^2 x + 11}$ ,  $f'(x) = \frac{1}{2\sqrt{\cos^2 x + 11}}$ .

Ans: False

Difficulty: Hard

Section: 2.6

5. If  $f(x) = x^7 \sqrt{x^3 + 6}$ , then  $f'(x) =$

A)  $7x^3 \sqrt{x^3 + 6} + \frac{3x^9}{2\sqrt{x^3 + 6}}$     D)  $\frac{3x^9}{2\sqrt{x^3 + 6}}$   
 B)  $7x^6 \sqrt{x^3 + 6} + \frac{3x^9}{\sqrt{x^3 + 6}}$     E)  $7x^6 \sqrt{x^3 + 6} + \frac{3x^9}{2\sqrt{x^3 + 6}}$   
 C)  $7x^6 \sqrt{x^3 + 6}$

Ans: E

Difficulty: Hard

Section: 2.6

6.  $y = \cot(\sin x)$ . Find  $dy/dx$ .

- A)  $\cos x \csc^2(\sin x)$  D)  $\sin x \csc^2(\cos x)$   
 B)  $-\cos x \csc^2(\sin x)$  E)  $-\sin x \csc^2(\sin x)$   
 C)  $-\sin x \csc^2(\cos x)$

Ans: B

Difficulty: Medium

Section: 2.6

7.  $y = x^4 \sin(5x)$ . Find  $dy/dx$ .

- A)  $5x^4 \cos(5x) - 4x^3 \sin(5x)$  D)  $5x^4 \cos(5x)$   
 B)  $5x^4 \cos(5x) + x^3 \sin(5x)$  E)  $4x^3 \sin(5x)$   
 C)  $5x^4 \cos(5x) + 4x^3 \sin(5x)$

Ans: C

Difficulty: Medium

Section: 2.6

8. If  $y = \frac{1 - \sin^2 x}{\cos x}$ ,  $\frac{dy}{dx} =$

- A)  $\cos x$  B)  $-\sin x$  C)  $-\cos x$  D)  $\sin x$  E)  $\sin x \cos x$

Ans: B

Difficulty: Easy

Section: 2.6

9. If  $y = \left( \frac{1 - x^2 \cos^2 x}{\sin x} \right)$ ,  $\frac{dy}{dx} =$  (Do not reduce any trig ratio identities.)

- A)  $\frac{2x \cos^2 x \sin x - 2x^2 \cos x \sin^2 x + \cos x - x^2 \cos^2 x}{\sin^2 x}$   
 B)  $\frac{-2x \cos^2 x \sin x + 2x^2 \cos x \sin^2 x - \cos x + x^2 \cos^3 x}{\sin^2 x}$   
 C)  $\frac{2x \cos^2 x \sin x + 2x^2 \cos x \sin^2 x - x^2 \cos^2 x}{\sin^2 x}$   
 D)  $\frac{2x \cos^2 x \sin x + 2x^2 \cos x \sin^2 x - \cos x - x^2 \cos^2 x}{\sin x}$   
 E)  $\frac{2x \cos^2 x \sin x + 2x^2 \sin^2 x - \cos x - x^2 \cos^2 x}{\sin^2 x}$

Ans: B

Difficulty: Hard

Section: 2.6

10. Answer true or false. If  $y = \cos(x^6)$ ,  $d^2y/dx^2 = -\cos(x^6)$ .

Ans: False

Difficulty: Easy

Section: 2.6

11. Answer true or false. If  $y = \sin 6x - \cos x^3$ ,  $d^2y/dx^2 = 18 \cos x$ .

Ans: False

Difficulty: Easy

Section: 2.6

12. Find an equation for the tangent line to the graph of  $y = x \sin x$  at  $x = \pi$ .

A)  $y = -x + \pi$

D)  $y = x + \pi$

B)  $y = x - \pi$

E)  $y = -\pi x + \pi^2$

C)  $y = \pi x - \pi^2$

Ans: E

Difficulty: Medium

Section: 2.6

13.  $y = \sin^3(\pi - 9\theta)$ . Find  $dy/d\theta$ .

A)  $-27 \cos(\pi - 9\theta) \sin^2(\pi - 9\theta)$

D)  $-\cos(\pi - 9\theta) \sin^2(\pi - 9\theta)$

B)  $-27 \cos(\pi + 9\theta) \sin^2(\pi + 9\theta)$

E)  $-27 \sin^2(\pi - 9\theta)$

C)  $-27 \cos^2(\pi - 9\theta) \sin(\pi - 9\theta)$

Ans: A

Difficulty: Medium

Section: 2.6

14. Use a graphing utility to obtain the graph of  $f(x) = x^8 \sqrt[4]{x}$ . Determine the slope of the tangent line to the graph at  $x = 0$ .

A)  $\frac{\sqrt{7}}{4}$     B) 0    C) 3    D) 16    E) 8

Ans: B

Difficulty: Medium

Section: 2.6

15. Find the value of the constant  $A$  in  $y = A \sin 3t$  so that  $d^2y/dt^2 + 7y = \sin 3t$ .

A)  $\frac{1}{2}$     B)  $\frac{1}{16}$     C)  $-\frac{1}{2}$     D) 2    E) 16

Ans: C

Difficulty: Hard

Section: 2.6

16. Answer true or false. Given  $f'(x) = 5x$  and  $g(x) = \sqrt{5x}$ , then  $F'(x) = 5x\sqrt{5x}$  if  $F(x) = f(g(x))$ .

Ans: False

Difficulty: Medium

Section: 2.6

17. Find  $f'(x)$  if  $f(x) = (x^5 - 11)^{10}$ .

A)  $10x^4(x^5 - 11)^9$

D)  $50x^4(x^5 - 11)^9$

B)  $5x^4(x^5 - 11)^9$

E)  $10(x^5 - 11)^9$

C)  $50(x^5 - 11)^9$

Ans: D

Difficulty: Medium

Section: 2.6

18. Find  $dy/dx$  if  $y = \cos(\sin x)$ .

A)  $-\cos x \sin(\cos x)$

D)  $-\cos x \sin(\sin x)$

B)  $-\sin(\sin x)$

E)  $\sin(\cos x)$

C)  $\cos x \sin(\sin x)$

Ans: D

Difficulty: Medium

Section: 2.6

19. Find  $f'(x)$  where  $f(x) = x^2(\sin 8x)^6$ .

Ans:  $f'(x) = 2x (\sin 8x)^6 + 48x^2 (\cos 8x)(\sin 8x)^5$

Difficulty: Medium

Section: 2.6

$$f(x) = \frac{5}{(x^2 - 2x + 5)^5}$$

20. Find  $f'(x)$  where

$$f'(x) = \frac{-25(2x-2)}{(x^2 - 2x + 5)^6}$$

Ans:

Difficulty: Medium

Section: 2.6

21. Find  $f'(x)$  where  $f(x) = \cos(\tan 4x)$ .

Ans:  $-4 \sin(\tan 4x) \sec^2 4x$

Difficulty: Medium

Section: 2.6

22. Find  $f'(\theta)$  where  $f(\theta) = (\theta + \sin 2\theta)^2$ .

Ans:  $2(\theta + \sin 2\theta)(1 + 2 \cos 2\theta)$

Difficulty: Medium

Section: 2.6

23. Find  $f'(\theta)$  where  $f(\theta) = \cos^2(6\theta^6 - \theta)^6$ .

Ans:  $-12(6\theta^6 - \theta)^5(36\theta^5 - 1)\cos(6\theta^6 - \theta)^6 \sin(6\theta^6 - \theta)^6$

OR:

$-12(6\theta^6 - \theta)^5(36\theta^5 - 1)\cos 2(6\theta^6 - \theta)^6$

Difficulty: Hard

Section: 2.6

24. Find  $f'\left(\frac{\pi}{20}\right)$  where  $f(x) = \sin^6 5x$ .

Ans:  $f'(x) = 30\sin^5 5x \cos 5x$

$f'\left(\frac{\pi}{20}\right) = \frac{15}{4}$

Difficulty: Medium

Section: 2.6

25. Find  $f'(x)$  where  $f(x) = \csc^4 4x$ .

Ans:  $-16 \csc^4 4x \cot 4x$

Difficulty: Medium

Section: 2.6

26. Find  $f'(x)$  where  $f(x) = \sec^2(7x - x^2)$ .

Ans:  $2(7 - 2x)\sec^2(7x - x^2)\tan(7x - x^2)$

Ans:

Difficulty: Medium

Section: 2.6

27. Find  $f'(x)$  where  $f(x) = (x^5 - 3)^7(x^5 + 2)^4$ .

Ans:  $35x^4(x^5 - 3)^6(x^5 + 2)^4 + 20x^4(x^5 + 2)^3(x^5 - 3)^7 = 5x^4(x^5 - 3)^6(x^5 + 2)^3(11x^5 + 2)$

Difficulty: Hard

Section: 2.6

28. Find  $\frac{dy}{dx}$  where  $y = (6x + 3)^6(2x + 3)^4$ .  
 Ans:  $36(6x + 3)^5(2x + 3)^4 + 8(6x + 3)^6(2x + 3)^3 = 4(6x + 3)^5(2x + 3)^3(30x + 33)$   
 Difficulty: Medium  
 Section: 2.6

29. Find  $\frac{dy}{dx}$  where  $y = \left(\frac{x+5}{x-5}\right)^2$   
 $\frac{20(x+5)}{(x-5)^3}$   
 Ans:  
 Difficulty: Medium  
 Section: 2.6

30. Find  $y'(\pi)$  where  $y = \left(\frac{1}{x} + \sin x\right)^{-1}$ .  
 Ans:  $\pi^2 + 1$   
 Difficulty: Hard  
 Section: 2.6

31. Find  $f'(t)$  where  $f(t) = \left(\frac{1}{t} + \frac{1}{t^2}\right)^4$ .  
 $f'(t) = 4\left(\frac{1}{t} + \frac{1}{t^2}\right)^3\left(-\frac{1}{t^2} - \frac{2}{t^3}\right)$   
 Ans:  
 Difficulty: Hard  
 Section: 2.6

32. Find equations for the tangent and normal lines to the graph of  $f(x) = \sin(16 - x^2)$  at  $x = 4$ .  
 Ans: Tangent:  $y = -8x + 32$   
 Normal:  $y = \frac{1}{8}x - \frac{1}{2}$   
 Difficulty: Hard  
 Section: 2.6

33. Find equations for the tangent and normal lines to the graph of  $f(x) = \cos(4 - x^2)$  at  $x = 2$ .  
 Ans: Tangent:  $y = 1$   
 Normal:  $x = 2$   
 Difficulty: Easy  
 Section: 2.6

34. Find equations for the tangent and normal lines to the graph of  $f(x) = x \cos 6x$  at  $x = \pi/6$ .

Ans: Tangent:  $y = -x$

Normal:  $y = x - \frac{\pi}{3}$

Difficulty: Medium

Section: 2.6

35. Find  $f'(x)$  where  $f(x) = (x^6 + 6x)^{11}$ .

Ans:  $11(x^6 + 6x)^{10}(6x^5 + 6)$

Difficulty: Medium

Section: 2.6

36. Find  $f'(x)$  where  $f(x) = \sqrt{x^6 + 7x + 8}$ .

$$f'(x) = \frac{6x^5 + 7}{2\sqrt{x^6 + 7x + 8}}$$

Ans:

Difficulty: Medium

Section: 2.6

37.  $y = x^3 \cos(3x)$ . Find  $dy/dx$ .

Ans:  $3x^2 \cos 3x - 3x^3 \sin 3x$

Difficulty: Medium

Section: 2.6

38. Find  $f'(x)$  if  $f(x) = \sqrt{x^8 + 7x^4 + 7}$ .

$$f'(x) = \frac{8x^7 + 28x^3}{2\sqrt{x^8 + 7x^4 + 7}}$$

Ans:

Difficulty: Medium

Section: 2.6

39. Find  $f'(x)$  if  $f(x) = \cos(x^6)$ .

Ans:  $-6x^5 \sin(x^6)$

Difficulty: Medium

Section: 2.6

40. Given that  $f(1) = 5$ ,  $f'(1) = 4$  and  $g(x) = (f(x))^{-4}$ , find  $g'(1)$ .

$$-\frac{16}{3,125}$$

Ans:

Difficulty: Hard

Section: 2.6

41. Find  $(f \circ g)'(3)$  if  $f'(3) = 5$ ,  $g(3) = 3$  and  $g'(3) = 2$ .

Ans:  $(f \circ g)'(x) = f'(g(x))g'(x) = 5 \cdot 2 = 10$

Difficulty: Medium

Section: 2.6

42. Find  $f'(x)$  if  $f(x) = \frac{1}{(x-1)^6}$ .

$$f(x) = \frac{-6}{(x-1)^7}$$

Ans:

Difficulty: Easy

Section: 2.6