

Test Bank for Linear Algebra 5th Poole

Sections 1.0 - 1.4

1. If $\mathbf{a} = [1, -2, 3]$, $\mathbf{b} = [4, 0, 1]$, $\mathbf{c} = [2, 1, -3]$, compute $2\mathbf{a} - 3\mathbf{b} + 4\mathbf{c}$.

ANSWER: $[-2, 0, -9]$

2. Simplify the following vector expression: $4\mathbf{u} - 2(\mathbf{v} + 3\mathbf{w}) + 6(\mathbf{w} - \frac{1}{2}\mathbf{u})$.

ANSWER: $\mathbf{a}$

3. Solve for the vector $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$: $2\mathbf{x} - \mathbf{a} = 3\mathbf{b} = 2(\mathbf{a} + \mathbf{b}) - (\mathbf{x} - \mathbf{b})$.

ANSWER: $\mathbf{x} = \left(\frac{3}{5}\right)\mathbf{a} + \left(\frac{2}{5}\right)\mathbf{b}$

4. Find values of the scalar k for which the following vectors are orthogonal.

$\mathbf{u} = [k, k, -2]$, $\mathbf{v} = [-2, k - 1, 5]$

ANSWER: $k = 5, -2$

5. If $\mathbf{u}$ is orthogonal to $\mathbf{v}$, then which of the following is also orthogonal to $\mathbf{v}$?

ANSWER: $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v}) = 0$

6. Find all values of k such that $d(\mathbf{a}, \mathbf{b}) = 6$, where $\mathbf{a} = [2, k, 1, -4]$ and $\mathbf{b} = [3, -1, 6, -3]$.

ANSWER: $k = 2, -4$

7. Suppose that the dot product of $\mathbf{u} = [u_1, u_2]$ and $\mathbf{v} = [v_1, v_2]$ in $\mathbb{R}^2$ were defined as $\mathbf{u} \cdot \mathbf{v} = 5u_1 v_1 + 2u_2 v_2$. Consider the following statements for vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and all scalars c .

a. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

b. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

c. $(c\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$

d. $\mathbf{u} \cdot \mathbf{u} \geq 0$ and $\mathbf{u} \cdot \mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}$

ANSWER: (a) true (b) true (c) true (d) true

8. Suppose that the dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ were defined as the product of the lengths of the vectors.

Which (if any) of the following statements would be true for all vectors $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$, and all scalars c ?

a. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

b. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$

c. $c(\mathbf{u}) \cdot \mathbf{v} = c(\mathbf{u} \cdot \mathbf{v})$

d. $\mathbf{u} \cdot \mathbf{u} \geq 0$ and $\mathbf{u} \cdot \mathbf{u} = 0$ if and only if $\mathbf{u} = \mathbf{0}$

ANSWER: (a) true (b) false (c) false (d) true

9. What is the vector represented by an arrow from the origin (1,2) to the point (4,5) in $\mathbb{R}^2$?

a. (3, 3)

b. (3, -3)

c. (4, 3)

d. (3, 4)

ANSWER: $\mathbf{a}$

10. Solve for $\mathbf{x}$ in vector equation: $\mathbf{x} - 2(1,2) = (3,4) - (2,3)$

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a. (5,6)

b. (3,5)

c. (7,8)

d. (1,2)

ANSWER: b11. Given $\mathbf{p} = [1, -2, 1]$, $\mathbf{q} = [4, -4, 7]$, find:a. $\mathbf{p} \cdot \mathbf{q}$ b. $\|\mathbf{p} - \mathbf{q}\|$ c. $\text{proj}_{\mathbf{q}}\mathbf{p}$ d. the cosine of the angle between $\mathbf{p}$ and $\mathbf{q}$ **ANSWER:** (a) 19

(b) 7

(c) $\left(\frac{19}{81}\right)\mathbf{q}$ (d) $\frac{19}{9\sqrt{6}}$ 12. Let $ABCDEF$ be a regular hexagon whose sides are of length 5. If $\vec{AB} = \mathbf{a}$ and $\vec{BC} = \mathbf{b}$, find the projection of $\vec{FA}$ along $\mathbf{a}$.**ANSWER:** $(5/2)\mathbf{a}$ 13. Find the distance between the planes $2x - y + z = 1$ and $-6x + 3y - 3z = 1$.**ANSWER:** $4/3\sqrt{6}$

14. Find the distance between the parallel lines.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \end{bmatrix} \text{ and } \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + t \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

ANSWER: $\sqrt{2}$ 15. Find the acute angle between the planes $2x + 2y + 2z = 3$ and $2x - 2y - z = 5$.**ANSWER:** 1.76 radians16. In a parallelogram $ABCD$ let $\vec{AB} = \mathbf{a}$, $\vec{AD} = \mathbf{b}$. Let M be the point of intersection of the diagonals. Express $\vec{MA}$, $\vec{MB}$, $\vec{MC}$, and $\vec{MD}$ as linear combinations of $\mathbf{a}$ and $\mathbf{b}$.**ANSWER:** $\vec{MA} = \frac{1}{2}(\mathbf{a} + \mathbf{b})$, $\vec{MD} = \frac{1}{2}(\mathbf{a} - \mathbf{b})$, $\vec{MC} = -\vec{MA}$, $\vec{MB} = -\vec{MD}$ 17. Find all solutions of $3x + 5 = 2$ in $\mathbb{Z}_7$, or show that there are no solutions.

a. 2

b. 4

c. 6

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d. 8

ANSWER: $x = 6$

18. Find all solutions of $7x = 1$ in $\mathbb{Z}_{11}$, or show that there are no solutions.

ANSWER: $x = 8$

19. Find the check digit that should be appended to the vector $\mathbf{u} = [2, 5, 6, 4, 5]$ in $\mathbb{Z}_7^5$ if the check vector is $\mathbf{c} = [1, 1, 1, 1, 1, 1]$.

ANSWER: 6

20. Let $\mathbf{u} = (2, 1)$ and $\mathbf{v} = (1, 3)$ be two vectors in $\mathbb{R}^2$. What is the coordinate grid determined by these vectors?

- a. A grid with lines of slope $\frac{1}{2}$ spaced 5 units apart and lines of slope 3 spaced 10 units apart.
- b. A grid with lines of slope $\frac{1}{2}$ spaced $\sqrt{5}$ units apart and lines of slope 3 spaced $\sqrt{10}$ units apart.
- c. A grid with lines of slope $\frac{1}{2}$ spaced 1 unit apart and lines of slope 3 spaced 3 units apart.
- d. A grid with lines of slope $\frac{1}{2}$ spaced 2 units apart and lines of slope 3 spaced 3 units apart.

ANSWER: b

21. . Given the vectors $\mathbf{a} = [-1, 2, 0]$, $\mathbf{b} = [-1, 1, 1]$, $\mathbf{c} = [3, 0, 1]$, the first component of the vector $\mathbf{v} = \mathbf{a} - 2\mathbf{b} + \frac{1}{2}\mathbf{c}$ is:

- a. -2
- b. 0
- c. 2
- d. 4

ANSWER: c

22. In a triangle ABC , let K be the midpoint of the side $\overrightarrow{BC}$, and let M be the midpoint of $\overrightarrow{AC}$.

Let $\mathbf{p} = \overrightarrow{AK}$ and $\mathbf{q} = \overrightarrow{BM}$. Then $\overrightarrow{AB}$ can be expressed in terms of $\mathbf{p}$ and $\mathbf{q}$ as:

- a. $\frac{2}{3}(\mathbf{p} + \mathbf{q})$
- b. $\frac{1}{3}\mathbf{p} - \frac{2}{3}\mathbf{p}$
- c. $\frac{3}{2}(\mathbf{p} - \mathbf{q})$
- d. $\frac{2}{3}(\mathbf{p} - \mathbf{q})$

ANSWER: d

23. Given $\mathbf{a} = [2, 3, 0]$, $\mathbf{b} = [0, -3, -2]$, then the orthogonal projection of $\mathbf{a} + \mathbf{b}$ along $[0, 1, 1]$ is:

- a. $[0, -2, -2]$
- b. $[2, 0, -2]$

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- c. $[0, 6, 0]$
- d. $[0, 2, -2]$

ANSWER: a

24. If $\mathbf{a} = [1, 3, z]$, $\mathbf{b} = [x, -6, 2]$, then $\mathbf{a}$ and $\mathbf{b}$ are collinear vectors if:

- a. $x = 2, z = 1$
- b. $x = -2, z = -1$
- c. $x = -1, z = 2$
- d. $x = 1, z = 2$

ANSWER: b

25. Suppose $\|\mathbf{p}\| = 3$ and $\|\mathbf{q}\| = 5$. Then the vectors $\mathbf{p} + k\mathbf{q}$ and $\mathbf{p} - k\mathbf{q}$ are orthogonal if:

- a. $k = -1$
- b. $k = \frac{5}{3}$
- c. $k = -\frac{5}{3}$
- d. $k = \frac{3}{5}$

ANSWER: d

26. Suppose that the dot product of two vectors in $\mathbb{R}^2$ were defined as the product of the lengths of the vectors. Which of the following would be false?

- a. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
- b. $\mathbf{u} \cdot \mathbf{u} \geq 0$
- c. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
- d. $\|\mathbf{u}\| = \sqrt{\mathbf{u} \cdot \mathbf{v}}$

ANSWER: c

27. If $\mathbf{u}_1$ and $\mathbf{u}_2$ are unit vectors and $\mathbf{a} = \mathbf{u}_1 + 2\mathbf{u}_2$ is perpendicular to $\mathbf{b} = 5\mathbf{u}_1 - 4\mathbf{u}_2$, then the angle between $\mathbf{u}_1$ and $\mathbf{u}_2$ is:

- a. $\frac{\pi}{6}$
- b. $\frac{\pi}{3}$
- c. $\frac{\pi}{2}$
- d. $\frac{2\pi}{3}$

ANSWER: b

28. Which of the following is not true for all vectors $\mathbf{u}, \mathbf{v}$?

- a. $\|\mathbf{u} - \mathbf{v}\| \leq \|\mathbf{u}\| - \|\mathbf{v}\|$

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b. $\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$

c. $\mathbf{u} \cdot \mathbf{v} = \frac{1}{4} \|\mathbf{u} + \mathbf{v}\|^2 - \frac{1}{4} \|\mathbf{u} - \mathbf{v}\|^2$

d. $|\mathbf{u} \cdot \mathbf{v}| \leq \|\mathbf{u}\| \|\mathbf{v}\|$

ANSWER: a

29. For all vectors $\mathbf{v}$ in $\mathbb{R}^n$, the nonzero vector $\mathbf{u}$ is orthogonal to:

a. $\text{proj}_{\mathbf{u}}(\mathbf{v})$

b. $\text{proj}_{\mathbf{v}}(\mathbf{u})$

c. $\mathbf{v} + \text{proj}_{\mathbf{u}}(\mathbf{v})$

d. $\mathbf{v} - \text{proj}_{\mathbf{u}}(\mathbf{v})$

ANSWER: d

30. Which of the following expresses the fact that the vectors $\mathbf{u}$ and $\mathbf{v}$ have the same length?

a. $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| - \|\mathbf{v}\|$

b. $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u}\| + \|\mathbf{v}\|$

c. $\mathbf{u} \cdot \mathbf{u} = \mathbf{v} \cdot \mathbf{v}$

d. $\frac{\mathbf{u}}{\|\mathbf{u}\|} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$

ANSWER: c

31. The vector form of the equation of the line in $\mathbb{R}^3$ through $P = (2, 0, -3)$ and parallel to the line with parametric equations:

$$x = -2 + t, y = 2t, z = 1 - \frac{1}{2}t \text{ is } \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

a. $\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \\ 1/2 \end{bmatrix}$

b. $\begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix}$

c. $\begin{bmatrix} 1 \\ 2 \\ -1/2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$

d. $\begin{bmatrix} 2 \\ 4 \\ -1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$

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ANSWER: b

32. The equation of the plane that is equidistant from the two planes $5x - 3y + z = -3$ and $10x - 6y + 2z = -7$ is:

- a. $5x - 3y + z = -6$
- b. $10x - 6y + 2z = -13$
- c. $5x - 3y + z = -6$
- d. $20x - 12y + 4z = -13$

ANSWER: d

33. Let L be the line with parametric equations $x = 1 + 2t$, $y = t$, $z = -1$. The distance from L to the point $P = (0, 1, 2)$ is:

- a. $\sqrt{11}$
- b. $3\sqrt{30}$
- c. $3\sqrt{(30/5)}$
- d. $54/5$

ANSWER: c

34. The distance between the two planes $2x - y + z = 1$ and $-4x + 2y - 2z = 1$ is:

- a. $\frac{3}{2\sqrt{6}}$
- b. $\frac{3}{4}$
- c. $\frac{3}{\sqrt{6}}$
- d. $\frac{9}{24}$

ANSWER: a

35. Which of the following equations has no solution?

- a. $7x = 1$ in $\mathbb{Z}_{11}$
- b. $3x + 5 = 2$ in $\mathbb{Z}_7$
- c. $6x = 2$ in $\mathbb{Z}_9$
- d. $7x + 1 = 2$ in $\mathbb{Z}_5$

ANSWER: c

36. What digit should be appended to the vector $u = [2, 5, 6, 4, 5]$ in $\mathbb{Z}_7^5$ if the check vector is:

$c = [1, 1, 1, 1, 1]$?

- a. 5
- b. 6
- c. 4

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d. 3

ANSWER: b

37. A certain check digit code uses the check vector $\mathbf{c} = [9, 8, 7, 6, 5, 4, 3, 2, 1]$ and appends a ninth digit to an eight-digit number $\mathbf{u} = [u_1, u_2, \dots, u_8]$ so that $\mathbf{u} \cdot \mathbf{c} = 0$ in $\mathbb{Z}_{10}$. If the first eight digits of a valid number in this scheme are 1372423, what is the ninth digit?

- a. 4
- b. 5
- c. 6
- d. 7

ANSWER: b

38. Which of the following is not equal to $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c}$ for all $\mathbf{a}, \mathbf{b}, \mathbf{c}$ in $\mathbb{R}^3$?

- a. $\mathbf{b} \cdot \mathbf{c} \times \mathbf{a}$
- b. $\mathbf{c} \cdot \mathbf{a} \times \mathbf{b}$
- c. $-\mathbf{b} \times \mathbf{a} \cdot \mathbf{c}$
- d. $\mathbf{b} \cdot \mathbf{a} \times \mathbf{c}$

ANSWER: d

39. Suppose that three vectors satisfy $\mathbf{a} + 2\mathbf{b} + \mathbf{c} = \mathbf{0}$. Which of the following is equal to $\mathbf{a} \times \mathbf{b}$?

- a. $\mathbf{c} \times \mathbf{b}$
- b. $\mathbf{a} \times \mathbf{c}$
- c. $\frac{1}{2}(\mathbf{a} \times \mathbf{c})$
- d. $\frac{1}{2}(\mathbf{c} \times \mathbf{a})$

ANSWER: d

40. The cosine of the angle between the planes $x + 2y - z + 1 = 0$ and $-x + 3z + 1 = 0$ is:

- a. $\frac{1}{\sqrt{60}}$
- b. $\frac{-2}{\sqrt{60}}$
- c. $\frac{-2}{\sqrt{15}}$
- d. $\frac{2}{\sqrt{15}}$

ANSWER: c

41. If $\mathbf{v}$ is any nonzero vector, then $6\mathbf{v}$ is a vector in the same direction as $\mathbf{v}$ with a length of 6 units.

- a. True
- b. False

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ANSWER: False

42. The vectors $[1, 2, 3]$ and $[k, 2k, 3k]$ have the same direction for all nonzero real numbers k ?

- a. True
- b. False

ANSWER: False

43. Let $\mathbf{u} = (3, 2)$ and $\mathbf{v} = (1, 4)$ be two vectors in $\mathbb{R}^2$. Which of the following best describes the coordinate grid determined by these vectors?

- a. A grid with lines of slope $2/3$ intersecting lines of slope 4 at 45-degree angles.
- b. A grid with lines of slope $2/3$ parallel to lines of slope 4.
- c. A grid with lines of slope $2/3$ perpendicular to lines of slope 4.
- d. A grid with lines of slope $2/3$ intersecting lines of slope 4 at 90-degree angles.

ANSWER: a

44. If $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w}$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{w}$.

- a. True
- b. False

ANSWER: False

45. The only real number c for which $[c, -2, 1]$ is orthogonal to $[2c, c, -4]$ is $c = 2$.

- a. True
- b. False

ANSWER: False

46. The distance between two points in $\mathbb{R}^n$ located by the vectors $\mathbf{u}$ and $\mathbf{v}$ is $\|\mathbf{u} - \mathbf{v}\|$.

- a. True
- b. False

ANSWER: True

47. The zero vector is orthogonal to every vector except itself.

- a. True
- b. False

ANSWER: False

48. The projection of a vector $\mathbf{v}$ onto a vector $\mathbf{u}$ is undefined if $\mathbf{v} = \mathbf{0}$.

- a. True
- b. False

ANSWER: False

49. The projection of a vector $\mathbf{v}$ onto a vector $\mathbf{u}$ is undefined if $\mathbf{u} = \mathbf{0}$.

- a. True
- b. False

ANSWER: True

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50. The projection of vector $\mathbf{v}$ onto vector $\mathbf{u}$ is equal to $\mathbf{v}$ if:

- a. $\mathbf{u}$ and $\mathbf{v}$ are orthogonal
- b. $\mathbf{u} = 0$
- c. $\mathbf{v} = 0$
- d. $\mathbf{u}$ and $\mathbf{v}$ are parallel

ANSWER: d

51. The length of the projection of vector $\mathbf{u}$ onto vector $\mathbf{v}$ is:

- a. less than or equal to length of $\mathbf{u}$
- b. greater than or equal to length of $\mathbf{u}$
- c. equal to length of $\mathbf{v}$
- d. equal to length of $\mathbf{u}$

ANSWER: a

52. The distance between the planes $\mathbf{n} \cdot \mathbf{x} = d_1$ and $\mathbf{n} \cdot \mathbf{x} = d_2$ is $|d_1 - d_2|$.

- a. True
- b. False

ANSWER: False

53. The area of the parallelogram with sides $\mathbf{a}$ and $\mathbf{b}$ with $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 4$ is 6. What is the angle between vectors $\mathbf{a}$ and $\mathbf{b}$?

- a. 30°
- b. 45°
- c. 60°
- d. 90°

ANSWER: a

54. $\mathbf{a} \cdot \mathbf{b} \times \mathbf{c} = 0$ if and only if the vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are coplanar.

- a. True
- b. False

ANSWER: True

55. The products $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$ and $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$ are equal if and only if $\mathbf{b} = 0$.

- a. True
- b. False

ANSWER: False

56. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are mutually orthogonal vectors in $\mathbb{R}^3$, then $(\mathbf{a} \times \mathbf{b} \cdot \mathbf{c})^2 = \|\mathbf{a}\|^2 \|\mathbf{b}\|^2 \|\mathbf{c}\|^2$.

- a. True
- b. False

ANSWER: True

57. If a parity check code is used in the transmission of a message consisting of a binary vector, then the total number of

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1's in the message will be even.

- a. True
- b. False

ANSWER: True

58. If $\mathbf{u} \cdot \mathbf{v} = 0$, then $\|\mathbf{u} + \mathbf{v}\| = \|\mathbf{u} - \mathbf{v}\|$.

- a. True
- b. False

ANSWER: True

59. For all vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in $\mathbb{R}^n$, $\mathbf{u} - (\mathbf{v} - \mathbf{w}) = \mathbf{u} + \mathbf{w} - \mathbf{v}$.

- a. True
- b. False

ANSWER: True

60. For all vectors $\mathbf{v}$ and scalars c , $\|c\mathbf{v}\| = c\|\mathbf{v}\|$.

- a. True
- b. False

ANSWER: True

61. The area of the parallelogram with sides $\mathbf{a}, \mathbf{b}$, is $\frac{1}{2} \|\mathbf{a} \times \mathbf{b}\|$

- a. True
- b. False

ANSWER: True

62. Given the vectors $\mathbf{a} = [-1, 2, 1]$, $\mathbf{b} = [3, 1, 1]$, $\mathbf{c} = [4, 3, 0]$, the orthogonal projection of $2\mathbf{a} - 3\mathbf{b}$ along $\mathbf{c}$ is:

- a. $\left[\frac{11}{3}, \frac{1}{3}, \frac{1}{3} \right]$
- b. $\left[-\frac{164}{25}, \frac{123}{25}, 0 \right]$
- c. $[-164, -123, 0]$
- d. $[11, -1, 1]$

ANSWER: a

63. The angle between $\mathbf{u} = [-1, 2, 0]$ and $\mathbf{e}_2 = [0, 1, 0]$ is:

- a. acute
- b. obtuse
- c. 90°
- d. 0°

ANSWER: a

64. Which of the following vectors $\mathbf{x}$ is collinear with $\mathbf{a} = [2, 1, -1]$ and satisfies the condition $\mathbf{a} \cdot \mathbf{x} = 3$?

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- a. $\left[\frac{1}{2}, 1, -1 \right]$
- b. $[1, 1, 0]$
- c. $\left[1, \frac{1}{2}, -\frac{1}{2} \right]$
- d. $[1, 0, -1]$

ANSWER: a

65. The distance from the point $Q = (1, 1)$ to the line $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ is:

- a. $\frac{3}{\sqrt{3}}$
- b. 3
- c. $\frac{3}{\sqrt{2}}$
- d. $\frac{3}{\sqrt{5}}$

ANSWER: a

66. The equation of the plane in $\mathbb{R}^3$ passing through the points $P = (1, 2, 0)$, $Q = (2, 1, 1)$ and parallel to the vector $\mathbf{v} = [3, 0, 1]$ is:

- a. $-x + 2y - 3z = 3$
- b. $x + 2y + 3z = -3$
- c. $x - 2y - 3z = 3$
- d. $x - 2y + 3z = -3$

ANSWER: a

67. Find a unit vector in $\mathbb{R}^3$ in the opposite direction to $\mathbf{v} = [1, 2, -2]$.

ANSWER: $-\frac{1}{3}[1, 2, -2]$

68. Find the vector parametric equation of the line in $\mathbb{R}^3$ that is perpendicular to the plane $2x - 3y + 7z - 4 = 0$ and which passes through the point $P = (1, -5, 7)$.

ANSWER: $\mathbf{x} = [1, -5, 7] + t[2, -3, 7]$

69. Show that the quadrilateral with vertices $A = (-3, 5, 6)$, $B = (1, -5, 7)$, $C = (8, -3, -1)$ and $D = (4, 7, -2)$ is a square.

ANSWER: Let points A, B, C, D be located by vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$. Then the lengths of the sides are $\|\mathbf{b} - \mathbf{a}\| = \|\mathbf{c} - \mathbf{d}\| = \|\mathbf{d} - \mathbf{a}\| = \|\mathbf{c} - \mathbf{b}\| = \sqrt{117}$, and dot products of each side with the adjacent ones are all zero.

70. Find a value of k so that the angle between the line $4x + ky = 20$ and the line $2x - 3y = -6$ is 45° .

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ANSWER: $k = -\frac{4}{5}, 20$

71. What is the distance of the point $P = (2, 3, -1)$ to the line of intersection of the planes $2x - 2y + z = -3$ and $3x - 2y + 2z = -17$?

ANSWER: $6/\sqrt{5}$

72. Let u_1 and u_2 be unit vectors, and let the angle between them be $\frac{\pi}{4}$ radians. What is the area of the parallelogram

whose diagonals are $\mathbf{d}_1 = 2\mathbf{u}_1 - \mathbf{u}_2$ and $\mathbf{d}_2 = 4\mathbf{u}_1 - 5\mathbf{u}_2$?

ANSWER: $\frac{3}{\sqrt{2}}$

73. Simplify the following expressions:

(a) $(\mathbf{a} + \mathbf{b} + \mathbf{c}) \times \mathbf{c} + (\mathbf{a} + \mathbf{b} + \mathbf{c}) \times \mathbf{b} + (\mathbf{b} - \mathbf{c}) \times \mathbf{a}$

(b) $(\mathbf{v} + 2\mathbf{w}) \cdot (\mathbf{w} + \mathbf{z}) \times (3\mathbf{z} + \mathbf{v})$

ANSWER: (a) $2(\mathbf{a} \times \mathbf{c})$
(b) $5\mathbf{w} \cdot \mathbf{z} \times \mathbf{x}$

74. Find a vector of length $\sqrt{3}$ that is orthogonal to both $\mathbf{a} = [2, 1, -3]$ and $\mathbf{b} = [1, -2, 1]$.

ANSWER: $\frac{5}{\sqrt{3}}[1, 1, 1]$

75. Find the orthogonal projection of $\mathbf{v} = [-1, 2, 1]$ onto the xz -plane.

ANSWER: $[-1, 0, 1]$

76. Show that if a vector $\mathbf{v}$ is orthogonal to two noncollinear vectors in a plane P , then $\mathbf{v}$ is orthogonal to every vector in P .

ANSWER: If $\mathbf{v} \cdot \mathbf{a} = \mathbf{v} \cdot \mathbf{b} = 0$, then $\mathbf{v} \cdot (c_1\mathbf{a} + c_2\mathbf{b}) = 0$.

77. Which vector in $\mathbb{R}^2$ has the same direction as the vector $(3, 4)$?

- a. $(6, 8)$
- b. $(4, 3)$
- c. $(-3, -4)$
- d. $(3, -4)$

ANSWER: a

78. Let us consider two vectors in $\mathbb{R}^2$: $\mathbf{a} = (2, 3)$ and $\mathbf{b} = (4, 1)$. Which is the linear combination of these vectors?

- a. $(6, 5)$
- b. $(3, 2)$
- c. $(8, 4)$
- d. $(10, 7)$

Sections 1.0 - 1.4

ANSWER: c

79. Which vector in $\mathbb{R}^2$ can be drawn as an arrow from the point (2,1) to the point (6,5)?

- a. (4,4)
- b. (8,6)
- c. (2,3)
- d. (6,4)

ANSWER: a

80. Find the normal form of the equation of the line passing through point (2,3) and having a slope of 4 in $\mathbb{R}^2$?

- a. $4(x-2) - (y-3) = 0$
- b. $4(x-2) + (y-3) = 0$
- c. $(x-2) - 4(y-3) = 0$
- d. $(x-2) + 4(y-3) = 0$

ANSWER: a

81. Find the normal form of the equation of the line passing through points (1,2) and (3,4) in $\mathbb{R}^2$?

- a. $2(x-1) + (y-2) = 0$
- b. $2(x-1) - (y-2) = 0$
- c. $(x-1) - 2(y-2) = 0$
- d. $(x-1) + 2(y-2) = 0$

ANSWER: b

82. Find the vector form of the equation of the line passing through points $(-2,1)$ and $(4,-3)$ in $\mathbb{R}^2$?

- a. $[x, y] = [-2, 1] + t[6, -4]$
- b. $[x, y] = [-2, 1] + t[3, 2]$
- c. $[x, y] = [4, -3] - t[6, -4]$
- d. $[x, y] = [4, -3] - t[3, 2]$

ANSWER: a

83. Which of the following equations is in the general form of a line in $\mathbb{R}^2$?

- a. $2x - 3y = 5$
- b. $x/2 + 2y = 3$
- c. $4x + 2y - 1 = 0$
- d. $y = 2x - 1$

ANSWER: c

Sections 1.0 - 1.4

84. Find the parametric equation of the line passing through point (2,3) and having direction vector = [4, 5] in $\mathbb{R}^2$?

- a. $x = 2 + 4t, y = 3 + 5t$
- b. $x = 4 + 2t, y = 5 + 3t$
- c. $x = 2 + 4t, y = 3 - 5t$
- d. $x = 2 - 4t, y = 3 + 5t$

ANSWER: a

85. Find the normal form of equation of the plane passing through the point (2,3,4) and having a normal vector (3,4,5) $\mathbb{R}^3$.

- a. $3(x-2) + 4(y-3) + 5(z-4) = 9$
- b. $3x + 4y + 5z - 33 = 0$
- c. $3x + 4y + 5z - 38 = 0$
- d. $3(x-2) + 4(y-3) + 5(z-4) = 12$

ANSWER: c

86. Find the vector form of equation of the plane passing through the point (2,3,4), (1,2,3) and (3,4,5) $\mathbb{R}^3$.

- a. $[x, y, z] = [1, 2, 3] + s[1, 1, 1] + t[1, 1, 0]$
- b. $[x, y, z] = [1, 2, 3] + s[1, 1, 0] + t[1, 0, 1]$
- c. $[x, y, z] = [2, 3, 4] + s[1, 1, -1] + t[-1, 1, 1]$
- d. $[x, y, z] = [3, 4, 5] + s[1, -1, 1] + t[1, 1, -1]$

ANSWER: a

87. Find the parametric form of equation of the plane passing through the point (1,2,3) and having a normal vector [2, 3, 4] in $\mathbb{R}^3$.

- a. $x = 1 - 2s - 3t, y = 2 + 3s + 2t, z = 3 - 4s - 4t$
- b. $x = 1 + 2s + 3t, y = 2 - 3s - 2t, z = 3 + 4s - 4t$
- c. $x = 1 - 2s + 3t, y = 2 + 3s - 2t, z = 3 - 4s + 4t$
- d. $x = 1 + 2s - 3t, y = 2 - 3s + 2t, z = 3 + 4s + 4t$

ANSWER: d

88. Find the distance of the point (4,5,6) from the plane $2x - 3y + 4z = 10$ in $\mathbb{R}^3$.

- a. $3/\sqrt{29}$
- b. $5/\sqrt{29}$
- c. $7/\sqrt{29}$
- d. $9/\sqrt{29}$

ANSWER: c

89. A plane in $\mathbb{R}^3$ has the parametric equation:

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a. $x = 2 + 3s - 2t$,

b. $y = 3 + 2s + t$,

c. $z = 5 - s - 3t$.

Which of the following statements is TRUE?

- a. The plane passes through the origin (0,0,0).
- b. The plane contains the line $x = 2, y = 3$.
- c. The plane is parallel to the vector [1,1,1].
- d. The plane intersects the x -axis at (2,0,0).

ANSWER: d

90. When finding the distance from a point to a plane in $\mathbb{R}^3$, what is the role of the normal vector of the plane?

- a. It determines the orientation of the plane.
- b. It determines the position of the plane.
- c. It determines the direction of the shortest distance.
- d. It determines the length of the shortest distance.

ANSWER: c

91. What is the method to compute the resultant force of two or more force vectors?

- a. Subtract the magnitude of the forces.
- b. Add the magnitude of the forces.
- c. Take the average of the magnitude of the forces
- d. Use the vector addition method.

ANSWER: d

92. A force vector $F = 25\text{N}$ at an angle of 30° to the horizontal acts on an object. What is the vertical component of the force?

- a. 12.5N
- b. 15N
- c. 20N
- d. 25N

ANSWER: a

93. What is the range of values for the Pearson correlation coefficient (r)?

- a. $[-1,1]$
- b. $[0,1]$
- c. $[-0.5,0.5]$
- d. $[0.5,1.5]$

ANSWER: a

94. Given two data sets $X = \{1,2,3,4\}$ and $Y = \{2,3,5,7\}$, what is the value of the Pearson correlation coefficient (r)?

- a. 0.94
- b. 0.96
- c. 0.98

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d. 0.99

ANSWER: b

95. A set of forces $F_1 = 3i + 4j$, $F_2 = 2i - 3j$, $F_3 = -i + 2j$ acts on an object. What is the resultant force (in vector form)?

a. $4i + 3j$

b. $6i - 2j$

c. $2i + 7j$

d. $4i - 5j$

ANSWER: a

96. A force vector $F = 20i - 15j + 30k$ acts on an object. What is the magnitude of the force in the xy -plane?

a. 20N

b. 25N

c. 15N

d. 40N

ANSWER: b

97. A correlation coefficient of 0.85 indicates that two data sets are:

a. Weakly correlated in opposite directions

b. Moderately correlated in the same direction

c. Strongly correlated in same direction

d. Not correlated at all

ANSWER: c

98. What does a scatter diagram with a cluster of points in the top right and bottom left corners indicate about the correlation between two variables?

a. Positive correlation

b. Negative correlation

c. No correlation

d. Non-linear correlation

ANSWER: a

99. What is the key difference between a scatter diagram that shows a strong correlation and one that shows weak correlation?

a. The direction of correlation

b. The amount of data points in the diagram

c. The closeness of the data points to a straight line

d. The scale of x - and y -axes

ANSWER: c

100. A data set has a mean of 20 and a Standard Deviation of 4. Which of the following values is most likely to be included in the data set?

a. 10

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- b. 15
- c. 35
- d. 40

ANSWER: b

101. What does a small SD in a data set indicate about the data?
- a. The data points are spread out over a large range of values.
 - b. The data points are clustered close together, with little variation.
 - c. The data points are randomly and evenly distributed.
 - d. The data points are all identical, with no variation.

ANSWER: b

102. Which statement best describes the correlation between two data sets with a correlation coefficient of -0.67 ?
- a. An increase in one variable is associated with a decrease in the other, but the relationship is weak.
 - b. An increase in one variable is associated with a decrease in the other, and the relationship is moderate.
 - c. An increase in one variable is associated with an increase in the other, but the relationship is weak.
 - d. No meaningful relationship exists between the two variables.

ANSWER: b

103. The SD of the data 2, 4, 6, 8, 10 is approximately:
- a. 2.5
 - b. 3
 - c. 3.5
 - d. 4

ANSWER: b

104. A researcher finds a correlation coefficient of 0.7 between the amount of time spent studying and exam scores. Which of the following conclusions can be drawn from this result?
- a. Studying longer causes higher exam scores.
 - b. Higher exam scores cause longer study sessions.
 - c. There is a strong linear relationship between studying and exam scores.
 - d. 70% of exam scores can be predicted by the amount of time spent studying.

ANSWER: c